

EE320
Semester 2/2020
H.W. #2

INSTRUCTION:

You can work individually or as a group (with the maximum of 4 people).

If you work in group, group work will be graded as a group and please do as the followings:

- (a.) you must hand in a full copy of the group work in each own handwriting.
- (b.) please state each member of your group and your name on your work.

Please save your file as “studentid_firstname_hw2.pdf”, for example, “4904641234_sunsiree_hw2.pdf” and hand in your work in the google classroom by Saturday, May 8, 2021, midnight.

RELATED CHAPTERS:

Chapter 7(Derivatives of More-Than-One Independent Variable Function)
Chapter 8(Optimization without Constraint: More-Than-One Independent Variable Cases)

DETAILS:

(A.) Please work on “PRACTICE PROBLEM SET 5 (Multivariate calculus: basis derivative and applications)”

Question 5 “Change in equilibrium price and quantity”

Question 7 “MRS”: Use the implicit function theorem to find $MRS_{x_1x_2}$

Question 8 “MRTS”

Suggested additional question that you could do on your own time to practice for the final exam. There is no need to hand in these questions.

Question 3, 4, 6.1, 9, 11, 12, 14, 15

(B.) Please work on “PRACTICE PROBLEM SET 6 (Multivariate calculus: Unconstrained optimization)”

Additional to what we have learned in class, please read cournot model in the supplementary section below.

Question 3 “Oligopolists and merger”

Question 5 “Multiproduct, firm collusion, and antitrust”

Question 11 “Profit function”

Suggested additional question that you could do on your own time to practice for the final exam. There is no need to hand in these questions.

Question 1, 4, 6, 8, 9, 10

(C.) Please consider a firm who produces two kinds of snack: A (Almond joy) and B (Banana Bread). Suppose this firm is the monopolist in both markets of A (Almond joy) and B (Banana Bread). Market demand for A and B are:

Market demand for A : $Q_A = 4(P_B - P_A)$

Market demand for B : $Q_B = 4(9 + P_A - 2P_B)$

Total cost of this firm is $TC = 2Q_A + 3Q_B + 100$. Find P_A, P_B, Q_A, Q_B that maximize firm's profit.

Supplementary section on “Cournot model of Oligopoly: Quantity setters”

Cournot model explains strategic interaction between the oligopolists in the market.

Imagine a market for one kind of good which can be produced by two firms: firm 1, producing Q_1 , and firm 2, producing Q_2 .

Suppose inverse market demand is $P(Q) = \alpha - Q$, where $Q = Q_1 + Q_2$.

Also suppose that each firm total cost is $C_i(Q_i) = cQ_i$, where $i = 1, 2$, $c < \alpha$.

Firm 1's profit function is:

$$\begin{aligned}\pi_1(Q_1, Q_2) &= P(Q)Q_1 - cQ_1 \\ \pi_1(Q_1, Q_2) &= P(Q_1 + Q_2)Q_1 - cQ_1 \\ \pi_1(Q_1, Q_2) &= (\alpha - Q_1 - Q_2)Q_1 - cQ_1\end{aligned}$$

Notice that firm 1's profit depends on firm 2's production quantity through inverse market demand function which depends on total quantity sold in the market.

Firm 2's profit function is:

$$\begin{aligned}\pi_2(Q_1, Q_2) &= P(Q)Q_2 - cQ_2 \\ \pi_2(Q_1, Q_2) &= P(Q_1 + Q_2)Q_2 - cQ_2 \\ \pi_2(Q_1, Q_2) &= (\alpha - Q_1 - Q_2)Q_2 - cQ_2\end{aligned}$$

Notice that firm 2's profit depends on firm 1's production quantity through inverse market demand function on total quantity sold in the market.

FOC for firm 1, given Q_2 , is:

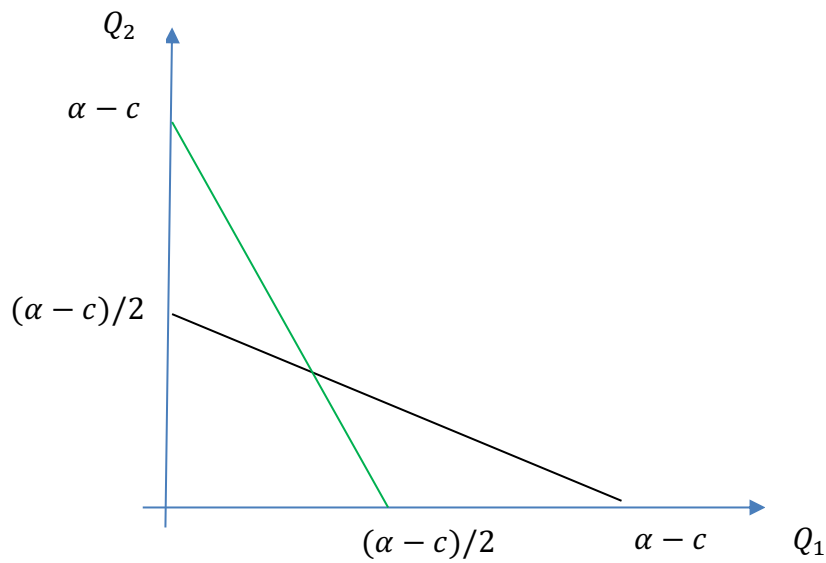
$$\frac{\partial \pi_1(Q_1, Q_2)}{\partial Q_1} = (\alpha - 2Q_1 - Q_2) - c = 0$$

That is, $Q_1^* = \frac{\alpha - c - Q_2}{2}$. This is called firm 1's best response function (the green line)

FOC for firm 2, given Q_1 , is:

$$\frac{\partial \pi_2(Q_1, Q_2)}{\partial Q_2} = (\alpha - Q_1 - 2Q_2) - c = 0$$

That is, $Q_2^* = \frac{\alpha - c - Q_1}{2}$. This is called firm 2's best response function (the black line).



The Nash equilibrium tells the exact market share of each firm in the market and happens when each firm's action is a best response to the other firm's action (the crossing point of the two best response functions). This can be found by:

Input Q_2^* in firm 1's best response function:

$$Q_1^* = \frac{\alpha - c - \left(\frac{\alpha - c - Q_1^*}{2}\right)}{2}$$

Hence, $Q_1^* = \frac{\alpha - c}{3}$, which gives $Q_2^* = \frac{\alpha - c}{3}$.