Multiple Regression Analysis with Qualitative Information:

1 Outline

- Describing qualitative information
- Using a single dummy independent variable
- Using dummy variables for multiple categories
- Interactions involving dummy variables
- A binary dependent variable (Y variable): The linear probability model

2 Describing Qualitative Information

- "Female" and "Married" are qualitative variable.
- We arbitarily assign a dummy variable to decribe them.

$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$

$$married = \begin{cases} 1 & \text{if married} \\ 0 & \text{otherwise (of if single)} \end{cases}$$

TABLE 7.1

A Partial Listing of the Data in WAGE1.RAW

person	wage	educ	exper	female	married
1	3.10	11	2	1	0
2	3.24	12	22	1	1
3	3.00	11	2	0	0
4	6.00	8	44	0	1
5	5.30	12	7	0	1
. . .	. . .	. . .	. . .	. . .	. . .
525	11.56	16	5	0	1
526	3.50	14	5	1	0

3 Models with a single dummy independent variable

Consider

$$wage = \beta_0 + \delta_0 female + \beta_1 educ + u.$$

where

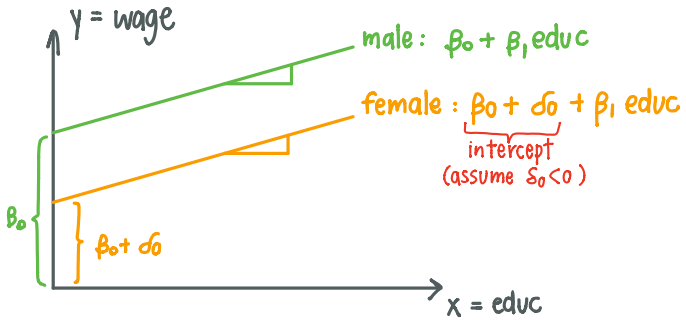
$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$

In this case, the δ_0 notation is used to highlight the interpretation of the parameters multiplying dummy variables. In other cases, we can use any notation that is the most convenient.

① $E(wage \mid female, educ) = E(\beta_0 + \delta_0 female + \beta_1 educ + u \mid female, educ)$
$= \beta_0 + \delta_0 female + \beta_1 educ + E(u \mid female, educ)$
$= \beta_0 + \delta_0 female + \beta_1 educ$ (0 (ass MLR1-4 holds))

② Thus ♀: $E(wage \mid female = 1, educ) = \beta_0 + \delta_0(1) + \beta_1 educ$
$= \beta_0 + \delta_0 + \beta_1 educ$
♂: $E(wage \mid female = 0, educ) = \beta_0 + \delta_0(0) + \beta_1 educ$
$= \beta_0 + \beta_1 educ$
$\delta_0 = E(wage \mid female = 1, educ) - E(wage \mid female = 0, educ)$
or $\delta_0 = E(wage \mid female, educ) - E(wage \mid male, educ)$

* given the same value of educ (same education level), δ_0 is the difference in the expected wage of females and males



↷ By the way we model this regression function "female" is going to give a constant impact on wage, regardless of the level of educ.

4 It is not possible to include all of the dummy alternatives in the same model (as long as there is the intercept in the model)

- If we include all alternatives of a dummy variable in the same model, we will face the "perfect collinearity" problem.

$$wage = \beta_0 x_0 + \delta_0 female + \beta_1 educ + \delta_1 male + u$$

(x_0: intercept $\times 1$; $female$: x_1; $educ$: x_2; $male$: x_3)

For example:

$$x_0 = x_1 + x_3$$
$$1 = female + male$$
$$female = male + 1$$

id.	female	male	x_0
1	1	0	1
2	1	0	1
3	0	1	1
4	0	1	1
⋮	⋮	⋮	⋮
99			

or

If there are "n" categories, we omit "1" category to avoid multi collinearity

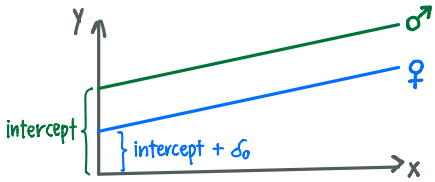
$$1 = winter + spring + summer + \cancel{fall}$$
$$winter = 1 - spring - summer - fall$$

$$winter = \begin{cases} 1 & \text{if winter} \\ 0 & \text{otherwise} \end{cases}$$
$$spring = \begin{cases} 1 & \text{if spring} \\ 0 & \text{otherwise} \end{cases}$$

id	winter	spring	summer	fall	x_0
1	1	0	0	0	1
2	0	1	0	0	1
3	0	1	0	0	1
4	0	0	1	0	1
⋮	⋮	⋮	⋮	⋮	⋮

in this case, male

- At least one alternative has to be dropped. We treat the dropped alternative as the "BASE GROUP" or "BASELINE" or "BENCHMARK GROUP".



female: $\begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise} \end{cases}$ male: $\begin{cases} 1 & \text{if male} \\ 0 & \text{otherwise} \end{cases}$

```
. regress lwage female male married educ exper
note: male omitted because of collinearity

      Source |       SS       df       MS              Number of obs =     526
-------------+------------------------------           F(  4,    521) =   75.27
       Model |  54.3265253     4  13.5816313            Prob > F      =  0.0000
    Residual |  94.0032262   521  .180428457           R-squared     =  0.3663
-------------+------------------------------           Adj R-squared =  0.3614
       Total |  148.329751   525  .28253286            Root MSE      =  .42477

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -.3251146   .0377061    -8.62   0.000    -.3991892    -.25104
        male |           0  (omitted)
     married |   .1380145   .0411197     3.36   0.001     .0572338    .2187953
        educ |   .0872644   .0071554    12.20   0.000     .0732075    .1013213
       exper |   .0076213   .0015314     4.98   0.000     .0046129    .0106297
       _cons |   .4690918   .1040575     4.51   0.000      .264668    .6735156
------------------------------------------------------------------------------
```

Female workers are expected to have less wage compared to male workers

5 Using dummy variables for multiple categories

Case 1 We can use many dummy variables in the same model

Consider a model which includes 2 dummy variables– $female$ and $married$.

{1 if female, 0 if otherwise} {1 if married, 0 otherwise}

$$\log(wage) = \beta_0 + \delta_0 female + \delta_1 married + \beta_1 educ + \beta_2 \exp er + \beta_3 \exp er^2 + \beta_4 tenure + \beta_5 tenure^2 + u.$$

```
regress lwage female married educ exper expersq tenure tenursq
```

Source	SS	df	MS		
Model	65.6482326	7	9.37831895	Number of obs	= 526
Residual	82.6815188	518	.159616832	F(7, 518)	= 58.76
				Prob > F	= 0.0000
Total	148.329751	525	.28253286	R-squared	= 0.4426
				Adj R-squared	= 0.4351
				Root MSE	= .39952

lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
female	-.2901838	.0361121	-8.04	0.000	-.3611279	-.2192396
married	.0529219	.0407561	1.30	0.195	-.0271456	.1329894
educ	.0791547	.0068003	11.64	0.000	.0657952	.0925143
exper	.0269535	.0053258	5.06	0.000	.0164907	.0374163
expersq	-.0005399	.0001122	-4.81	0.000	-.0007603	-.0003196
tenure	.0312962	.0068482	4.57	0.000	.0178426	.0447499
tenursq	-.0005744	.0002347	-2.45	0.015	-.0010355	-.0001134
_cons	.4177837	.0988662	4.23	0.000	.2235557	.6120116

($\hat{\beta}$)

Comments:

1) δ_0 measure the expected difference between female & male workers given the same marrital status and other factors

$$\frac{\partial \log(wage)}{\partial female} = \frac{\frac{1}{wage}\, d\,wage}{\partial female} = -0.29$$

$$\frac{100 \cdot \frac{1}{wage}\, d\,wage}{\partial female} = 100\,(-0.29)$$

$$\frac{\% \Delta wage}{\partial female} = 29.02\,\%$$

- female workers are expected to earn less than male workers by 29.02 %, holding other factors the same

2) δ_1 measure the impact of be married (marriage premium)
But since $|t| < 1.96$ or $p > 0.05$, we do not reject Ho of no impact

	♀	♂
marr	marrfem	marrmale
sing	singfem	singmale

Basecase

Consider a model which includes dummy variables for each gender/marital status combination– $marrmale$, $marrfem$ and $singfem$. (or singmale ↜ used as the basecase)

$$\log(wage) = \beta_0 + \delta_0 marrmale + \delta_1 marrfem + \delta_3 singfem + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + \beta_4 tenure + \beta_5 tenure^2 + u. \quad (8.1)$$

```
regress lwage marrmale marrfem singfem educ exper expersq tenure tenursq
```

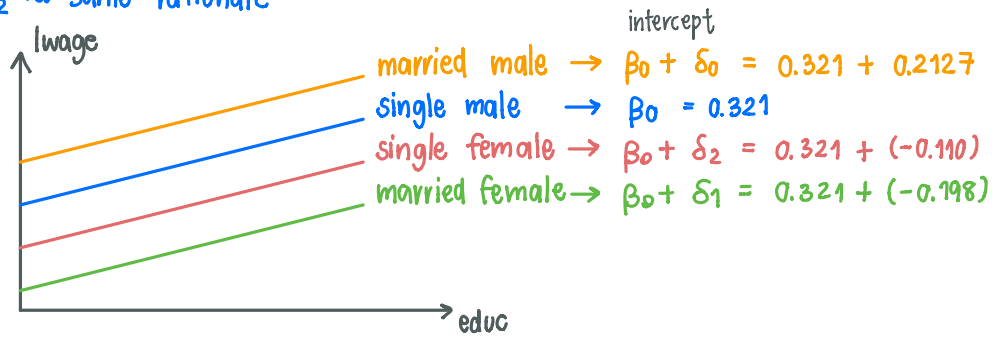
Source	SS	df	MS		
Model	68.3617623	8	8.54522029	Number of obs	= 526
Residual	79.9679891	517	.154676961	F(8, 517)	= 55.25
				Prob > F	= 0.0000
				R-squared	= 0.4609
				Adj R-squared	= 0.4525
Total	148.329751	525	.28253286	Root MSE	= .39329

	lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
δ_0	marrmale	.2126757	.0553572	3.84	0.000	.103923	.3214284
δ_1	marrfem	-.1982676	.0578355	-3.43	0.001	-.311889	-.0846462
δ_2	singfem	-.1103502	.0557421	-1.98	0.048	-.219859	-.0008414
$\hat{\beta}$	educ	.0789103	.0066945	11.79	0.000	.0657585	.092062
	exper	.0268006	.0052428	5.11	0.000	.0165007	.0371005
	expersq	-.0005352	.0001104	-4.85	0.000	-.0007522	-.0003183
	tenure	.0290875	.006762	4.30	0.000	.0158031	.0423719
	tenursq	-.0005331	.0002312	-2.31	0.022	-.0009874	-.0000789
	_cons	.3213781	.100009	3.21	0.001	.1249041	.5178521

The regression is not the same as the previous one. It used "Single Male" as the base group. (The previous one use male & single as 2 base groups)

Comments:

- δ_0 measures the expected diff in wage of married male as compared with single males, holding other factors constant
- δ_1 measures the expected diff in wage of married female as compared with single males, holding other factors constant
- δ_2 ↜ same rationale



Case 2 We can use dummy variables to represent multiple categories of a variable Consider the relationship between law school rankings and starting salaries

$$\begin{aligned}\log(salary) &= \beta_0 + \delta_0 top10 + \delta_1 r11\_25 + \delta_3 r26\_40 + \delta_4 r41\_60 + \beta_1 LSAT \\ &\quad + \beta_2 GPA + \beta_3 \log(libvol) + \beta_4 \log(cost) + u.\end{aligned}$$

where $top10$, $r11\_25$, $r26\_40$, $r41\_60$ would be equal to 1 when the variable $rank$ falls into the appropriate range.

** Rank below 60 would be the base case.

```
. regress lsalary top10 r11_25 r26_40 r41_60 LSAT GPA llibvol lcost
```

Source	SS	df	MS		
Model	9.16538532	8	1.14567316	Number of obs	= 136
Residual	1.2109665	127	.009535169	F(8, 127)	= 120.15
				Prob > F	= 0.0000
Total	10.3763518	135	.076861865	R-squared	= 0.8833
				Adj R-squared	= 0.8759
				Root MSE	= .09765

lsalary	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
top10 ✓	.5393428	.053542	10.07	0.000	.4333927	.6452928
r11_25 ✓	.4716199	.0390921	12.06	0.000	.3942637	.548976
r26_40 ✓	.2790977	.0346972	8.04	0.000	.2104383	.3477571
r41_60 ✓	.182382	.0283098	6.44	0.000	.126362	.238402
LSAT	.0060482	.0034919	1.73	0.086	-.0008616	.012958
GPA	.1305893	.0818678	1.60	0.113	-.0314122	.2925908
llibvol	.0725522	.0289213	2.51	0.013	.0153221	.1297824
lcost	.0249169	.0283224	0.88	0.381	-.031128	.0809619
_cons	8.363103	.4457314	18.76	0.000	7.481081	9.245125

the baseline is ranking 61th and worse

Comments:

☆ In many cases the "range of value" serve as a better explanatory variable than the "value" itself. e.g. age may explain the model better if split into generations young 0-15, gen-z 16-19, etc.

1) So measure the difference in expected log(salary) of a law-school graduate from a top 10 university compared to expected log(salary) of those who graduated from the school rank 61th and worse.

2) δ_1 ⇝ use the same rationale

rank	top 10	r11-15	r26-40	etc
1	1	0	0	
2	1	0	0	
3	1	0	0	
⋮	1	⋮	⋮	
10	1	0	0	
11	0	1	0	
12	0	1	0	
⋮	⋮		⋮	
25	0	1	0	
26	0	0	1	
⋮	⋮	⋮	⋮	
40	0	0	1	
⋮	⋮	⋮	⋮	