

Hint: ข้อสอบ ปี 1.5

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$$\text{จาก } \sigma_t^2 = \omega + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2 \Rightarrow \text{GARCH}(1,1)$$

$$E[a_t^4] = m_4 \Rightarrow \text{ข้อสอบ 2521} \quad \frac{E[a_t^4]}{E[a_t^2]^2} > 3$$

$$E[a_t^4] = E\left[E[a_t^4 | \mathcal{F}_{t-1}]\right]$$

$$= E\left[3 \cdot \left(E[a_t^2 | \mathcal{F}_{t-1}]\right)^2\right]$$

↳ conditional variance

$$\underline{E[a_t^4]} = E\left[3 \cdot \sigma_t^4\right] = \underline{3 E[\sigma_t^4]} = 3 \sigma_t^4$$

property 1:

$$E[a_t^4] = 3 E[\sigma_t^4]$$

$$E[a_t^4] = 3 \left[E\left[\omega + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2\right]^2 \right]$$

$$m_4 = 3 \left[E \left[\omega^2 + d^2 a_{t-1}^4 + B^2 b_{t-1}^4 + 2\omega \left[d a_{t-1}^2 + B b_{t-1}^2 \right] + 2dB a_{t-1}^2 b_{t-1}^2 \right] \right] \quad \text{--- (1)}$$

$$m_4 = 3 \left[E[\cdot] \right]$$

$$\Rightarrow E[a_{t-1}^2] = E[b_{t-1}^2] = \frac{\omega}{1 - (d+B)} = \text{Unconditional variance of } a_t$$

$\downarrow = m_2$

$$E[a_{t-1}^4] = E[a_t^4] = m_4$$

$$E[b_{t-1}^4] = \frac{m_4}{3}$$

same property 1

$$E[a_{t-1}^2 \cdot b_{t-1}^2] = \frac{m_4}{3}$$

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$$m_4 = \frac{3\omega^2(1+d+B)}{1 - (d+B)^2 - 2d^2}$$

$$\therefore m_4 = 3 \left[1 - (d+B)^2 \right]$$

$$\overline{(m_2)^2} = \frac{1 - (d+B)^2 - 2a^2}{\#}$$