Session 4-5 Theory of Risk and Return Exercise:

1. The following information is provided for a stock market:

	μ_j	β_j
Asset 1	6.6%	0.4
Asset 2	9.8%	1.2
Asset 3	12.2%	1.8

Notation: μ_j =expected rate of return on asset j; β_j =beta-coefficient for asset j, j = 1, 2, 3. In the context of the Capital Asset Pricing Model (CAPM), define the 'beta-coefficient', β_j, corresponding to asset j. Discuss how assets' beta-coefficients should be interpreted and explain how their values can be obtained in practice.

2. The following information is provided for a stock market:

	σ_j	ρ_{jM}
Security A	50%	0.6
Security B	60%	−0.2
Market Portfolio	20%	1.0

Notation: σ_j = standard deviation of the rate of return on asset j = A and j = B; ρ_{jM} = correlation coefficient between the return on asset j and the return on the market portfolio. The mean rate of return on the market portfolio is 8% and the risk-free rate of return is 5%.

- In the Capital Asset Pricing Model, explain what is meant by the beta coefficient, β_j, for a security. Calculate the beta coefficients for the two securities from the given information.
- You are told that the mean rates of return for securities A and B are 7.5% and 4.6% respectively. What would you infer from this information in the context of the Capital Asset Pricing Model?

3. The volatility of a stock price is 30% per annum. What is the standard deviation of the percentage price change in one week?

4. The volatility of an asset is 25% per annum. What is the standard deviation of the percentage price change in one trading day? Assuming a normal distribution, estimate 95% confidence limits for the percentage price change in one day.

5. Why do traders assume 252 rather than 365 days in a year when using volatilities?

6. Suppose that observations on an exchange rate at the end of the last 11 days have been 0.7000, 0.7010, 0.7070, 0.6999, 0.6970, 0.7003, 0.6951, 0.6953, 0.6934, 0.6923, 0.6922. Estimate the daily volatility with the theoretical formula and simplified version.

7. The number of visitors to a website follows the power law with $\alpha = 2$. Suppose that 1 % of sites get 500 or more visitors per day. What percentage of sites get (a) 1000 and (b) 2000 or more visitors per day?

8. Assume that S&P 500 at close of trading yesterday was 1,040 and the daily volatility of the index was estimated as 1% per day at that time. The parameters in a GARCH(1,1) model are $\omega = 0.000002$, $\alpha = 0.06$, and $\beta = 0.92$. If the level of the index at close of trading today is 1,060, what is the new volatility estimate?

9. The parameters of a GARCH(1,1) model are estimated as $\omega = 0.000004$, $\alpha = 0.05$, and $\beta = 0.92$. What is the long-run average volatility? If the current volatility is 20% per year, what is the expected volatility in 20 days?

10. Suppose that GARCH(1,1) parameters have been estimated as $\omega = 0.000003$, $\alpha = 0.04$, and $\beta = 0.94$. The current daily volatility is estimated to be 1%. Estimate the daily volatility in 30 days.

11. Suppose that GARCH(1,1) parameters have been estimated as $\omega = 0.000002$, $\alpha = 0.04$, and $\beta = 0.94$. The current daily volatility is estimated to be 1.3%. Estimate the volatility per annum that should be used to price a 20-day option.

12. Suppose that observations on a stock price (in US dollars) at the end of each of 15 consecutive weeks are as follows: 30.2, 32.0, 31.1, 30.1, 30.2, 30.3, 30.6, 33.0, 32.9, 33.0, 33.5, 33.5, 33.7, 33.5, 33.2 Estimate the stock price volatility.

13. Suppose that the price of gold at close of trading yesterday was $300 and its volatility was estimated as 1.3% per day. The price at the close of trading today is $298. Update the volatility estimate using the GARCH(1,1) model with $\omega = 0.000002$, $\alpha = 0.04$, and $\beta = 0.94$.

14. Suppose Portfolio A has an expected return of 8%, volatility of 20%, and beta of 0.5. Suppose the market has an expected return of 10% and volatility of 25%. Finally, suppose the risk-free rate is 5%. What is Jensen's alpha for Portfolio A?

15. You want to evaluate three mutual funds using the Sharpe measure for performance evaluation. The risk-free return during the sample period is 6%. The average returns, standard deviations and betas for the three funds are given below, as is the data for the S&P 500 index.

	Average Return	Standard. Deviation	Beta
Fund A	24%	30%	1.5
Fund B	12%	10%	0.5
Fund C	22%	20%	1.0
S&P 500	18%	16%	1.0

The fund with the highest Sharpe measure is _____.

16. You want to evaluate three mutual funds using the Sharpe measure for performance evaluation. The risk-free return during the sample period is 4%. The average returns, standard deviations and betas for the three funds are given below, as is the data for the S&P 500 index.

	Average Return	Standard. Deviation	Beta
Fund A	18%	38%	1.6
Fund B	15%	27%	1.3
Fund C	11%	24%	1.0
S&P 500	10%	22%	1.0

The fund with the highest Sharpe measure is _____.

17. You want to evaluate three mutual funds using the Sharpe measure for performance evaluation. The risk-free return during the sample period is 5%. The average returns, standard deviations and betas for the three funds are given below, as is the data for the S&P 500 index.

	Average Return	Standard. Deviation	Beta
Fund A	23%	30%	1.3
Fund B	20%	19%	1.2
Fund C	19%	17%	1.1
S&P 500	18%	15%	1.0

The investment with the highest Sharpe measure is _____.

18. You want to evaluate three mutual funds using the Treynor measure for performance evaluation. The risk-free return during the sample period is 6%. The average returns, standard deviations, and betas for the three funds are given below, in addition to information regarding the S&P 500 index.

	Average Return	Standard. Deviation	Beta
Fund A	13%	10%	0.5
Fund B	19%	20%	1.0
Fund C	25%	30%	1.5
S&P 500	18%	16%	1.0

The fund with the highest Treynor measure is _____.

19. You want to evaluate three mutual funds using the Jensen measure for performance evaluation. The risk-free return during the sample period is 6%, and the average return on the market portfolio is 18%. The average returns, standard deviations, and betas for the three funds are given below.

	Average Return	Standard. Deviation	Beta
Fund A	17.6%	10%	1.2
Fund B	17.5%	20%	1.0
Fund C	17.4%	30%	0.8

The fund with the highest Jensen measure is _____.

20. Suppose you purchase one share of the stock of Volatile Engineering Corporation at the beginning of year 1 for $36. At the end of year 1, you receive a $2 dividend, and buy one more share for $30. At the end of year 2, you receive total dividends of $4 (i.e., $2 for each share), and sell the shares for $36.45 each. The time-weighted return on your investment is _____.

21. Suppose you purchase one share of the stock of Volatile Engineering Corporation at the beginning of year 1 for $36. At the end of year 1, you receive a $2 dividend, and buy one more share for $30. At the end of year 2, you receive total dividends of $4 (i.e., $2 for each share), and sell the shares for $36.45 each. The dollar-weighted return on your investment is _____.

22. Suppose you purchase one share of the stock of Cereal Correlation Company at the beginning of year 1 for $50. At the end of year 1, you receive a $1 dividend, and buy one more share for $72. At the end of year 2, you receive total dividends of $2 (i.e., $1 for each share), and sell the shares for $67.20 each. The time-weighted return on your investment is _____.

23. Suppose you purchase one share of the stock of Cereal Correlation Company at the beginning of year 1 for $50. At the end of year 1, you receive a $1 dividend, and buy one more share for $72. At the end of year 2, you receive total dividends of $2 (i.e., $1 for each share), and sell the shares for $67.20 each. The dollar-weighted return on your investment is _____.