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1.

1.1) $m_{01} = \dfrac{\delta U'(C_1)}{U'(C_0)} = \delta\left(\dfrac{C_0}{C_1}\right) = \delta e^{\ln\left(\frac{C_0}{C_1}\right)}$

$$\frac{1}{R_f} = E(m_{01})$$

$$\frac{1}{R_f} = \delta E\left[e^{\ln\left(\frac{C_0}{C_1}\right)}\right]$$

$$\frac{1}{R_f} = \delta \quad e^{\mu_c + \frac{1}{2}\sigma_c^2}$$

$$-1 \ln(R_f) = \ln(\delta) - \mu_c + \frac{1}{2}\sigma_c^2$$

$$\ln(R_f) = -\ln(\delta) + \mu_c - \frac{1}{2}\sigma_c^2$$

$$\frac{1}{R_f} = \delta\left(\frac{C_0}{C_1}\right)$$

$$R_f = \frac{C_0}{\delta C_1}$$

$$\delta C_1 = \frac{C_0}{R_f}$$

R_f has negative relationship with C_1, when R_f increase, C_1 is decreased.

1.2) $U'(C_0) = R_f \delta E(U'(C_1))$

$$\frac{1}{C_0} = R_f \delta E\left[\frac{1}{C_1}\right]$$

$$\frac{1}{R_f} = \delta E\left[\frac{C_0}{C_1}\right]$$

$$R_f = \frac{1}{\delta}\left(\frac{C_1}{C_0}\right)$$

$$\frac{\partial R_f}{\partial\left(\frac{C_1}{C_0}\right)} = \frac{1}{\delta} \quad ; \quad \delta = \frac{C_1/C_0}{R_f}$$

$$\frac{\partial \frac{C_1}{C_0} / {C_1/C_0}}{\partial R_f / R_f} = \frac{C_1/\cancel{C_0}}{\cancel{R_f}} \cdot \frac{\cancel{R_f}}{\frac{C_1}{\cancel{C_0}}}$$

$$\varepsilon = 1$$

$\varepsilon = 1$, A higher interest rate reduces second period consumption one - for - one.

The income and substitution effect off-set each other.

1.3) $$1 = E[m_{01} R_i]$$

$$1 = E[m_{01}]E[R_i] + \mathrm{Cov}[m_{01}, R_i]$$

$$\frac{1}{E[m_{01}]} = E[R_i] + \frac{\mathrm{cov}[m_{01} R_i]}{E[m_{01}]}$$

$$R_f = E[R_i] + \frac{\mathrm{cov}[m_{01} R_i]}{E[m_{01}]}$$

$$E[R_i] = R_f - \frac{\mathrm{cov}[m_{01} R_i]}{E[m_{01}]}$$

$$= R_f - \frac{\mathrm{cov}[u'(c_1), R_i]}{E[u'(c_1)]}$$

An asset that tend to pay high returns when consumption is high has $\mathrm{cov}[u'(c_1), R_i] < 0$ and will have $E[R_i] > R_f$.

1.4) $$\left| \frac{E[R_i] - R_f}{\sigma_{R_i}} \right| \leq \frac{\sigma_{m_{01}}}{E[m_{01}]} = \sigma_{m_{01}} R_f$$

From 1.1) $$m_{01} = \delta e^{\ln\left(\frac{c_0}{c_1}\right)}$$

$$\frac{\sigma_{m_{01}}}{E[m_{01}]} = \frac{\sqrt{\mathrm{Var}[e^{\ln(c_0/c_1)}]}}{E[e^{\ln(c_0/c_1)}]}$$

$$= \frac{\sqrt{E[e^{2\ln(c_0/c_1)}] - E[e^{\ln(c_0/c_1)}]^2}}{E[e^{\ln(c_0/c_1)}]}$$

$$= \sqrt{\frac{E[e^{2\ln(c_0/c_1)}]}{E[e^{\ln(c_0/c_1)}]^2} - 1}$$

$$= \sqrt{e^{2\mu_c + 2\sigma_c^2} / e^{2\mu_c + \sigma_c^2} - 1}$$

$$= \sqrt{e^{\sigma_c^2} - 1}$$

$$\approx \sigma_c^2$$