

Quiz 4

1. Find

$$\int \left[x \sin(2 - 3x^2) + \frac{\sin(x)}{\sqrt{\cos(x)}} \right] dx.$$

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Solution

Consider

$$\int \left[x \sin(2 - 3x^2) + \frac{\sin(x)}{\sqrt{\cos(x)}} \right] dx = \int x \sin(2 - 3x^2) dx + \int \frac{\sin(x)}{\sqrt{\cos(x)}} dx.$$

We can apply substitution technique for each of the terms $\int x \sin(2 - 3x^2) dx$ and $\int \frac{\sin(x)}{\sqrt{\cos(x)}} dx$ separately.

(I) Let $u = 2 - 3x^2$. Then $du = -6x dx$ or $dx = \frac{1}{-6x} du$.

$$\begin{aligned} \int x \sin(2 - 3x^2) dx &= \int x \sin(u) \left[\frac{1}{-6x} \right] du \\ &= -\frac{1}{6} \int \sin(u) du \\ &= -\frac{1}{6} [-\cos(u)] + C \\ &= \frac{1}{6} [\cos(2 - 3x^2)] + C \end{aligned}$$

(II) Let $u = \cos(x)$. Then $du = -\sin(x) dx$ or $dx = \frac{1}{-\sin(x)} du$.

$$\begin{aligned} \int \frac{\sin(x)}{\sqrt{\cos(x)}} dx &= \int \frac{\sin(x)}{u^{1/2}} \frac{1}{-\sin(x)} du \\ &= -\int u^{-1/2} du \\ &= -\frac{u^{1/2}}{1/2} + C \\ &= -2[\cos(x)]^{1/2} + C \end{aligned}$$

From (I) and (II),

$$\int \left[x \sin(2 - 3x^2) + \frac{\sin(x)}{\sqrt{\cos(x)}} \right] dx = \frac{1}{6} \cos(2 - 3x^2) - 2[\cos(x)]^{1/2} + C$$

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