Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$ TSS	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$ RSS		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n}(X_i)^2 = 5,564$	$\sum_{i=1}^{n} X_iY_i = 7,524$	$\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n}(Y_i - \bar{Y})^2 = 882.97$	
$\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y}) = -174.20$		$\sum_{i=1}^{n}\hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

$$\hat{\beta}_2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_1 - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\hat{\beta}_2 = \frac{-174.20}{1098.8} = -0.159$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$\hat{\beta}_1 = 21.03 - (-0.159)\, 12.20$$

$$= 22.9698$$

$$\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

$$\hat{Y} = 22.9698 + (-0.159) X_i$$

$$\hat{Y} = 22.9698 - 0.159 X_i$$

$\hat{\beta}_1 = 22.9698$ mean if $X = 0$, $Y = 22.9698$

$\hat{\beta}_2 = -0.159$ mean if X increase 1 unit, Y decrease 0.159 unit

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

$RSS = \sum (Y_i - \hat{Y}_i)^2$ or $\sum \hat{\mu}^2$

$ESS = \sum (\hat{Y} - \bar{Y})^2$

$TSS = \sum (Y_i - \bar{Y})^2$

$TSS = RSS + ESS$

b) $r^2 = 1 - \frac{RSS}{TSS}$ or $\frac{ESS}{TSS}$

$r^2 = 1 - \frac{873.14}{882.97}$

$r^2 = 0.0111$

total variation in Y explained by the regression model 1.11%.

c) $E(Y | X_i = 5)$

$\hat{Y} = 22.9698 - 0.159 X_i$

$= 22.9698 - 0.159(5)$

$\hat{Y}_i = 22.173$

If $X_i = 5$, I expected $\hat{Y}_i = 22.173$

d) Find the estimators of $\operatorname{var}(u_i)$, $\operatorname{var}(\hat{\beta}_1)$ and $\operatorname{var}(\hat{\beta}_2)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

d) $\operatorname{Var}(u_i) = \hat{\sigma}^2 = \dfrac{\sum \hat{u}_i^2}{n-2}$

$$\sigma^2 = \frac{873.14}{30-2} = \boxed{31.1836}$$

$$\operatorname{var}(\hat{\beta}_1) = \frac{\sum X_i^2}{n\sum(X_i-\bar{X})^2}\cdot\sigma^2 =$$

$$= \frac{5564}{30\,(1098.8)}\cdot 31.1836$$

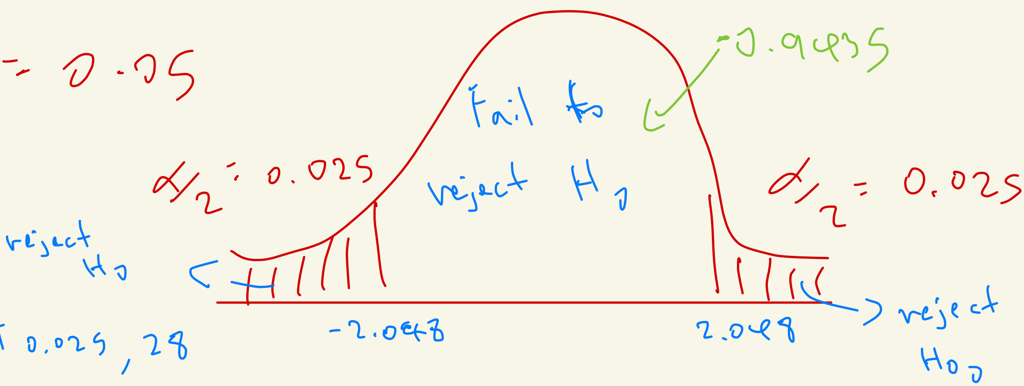
$$= \boxed{5.2635}$$

$$\operatorname{Var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum(X_i-\bar{X})^2} = \frac{31.1836}{1098.8} = \boxed{0.0284}$$

e) $H_0 : \beta_2 = 0$ (X no influence to Y)

$H_1 : \beta_2 \neq 0$ (X Influence to Y)

$\alpha = 0.05$



$t_{0.025,\,28}$

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{se\,\hat{\beta}_2} = \frac{-0.159 - 0}{\sqrt{0.0284}}$$

$$= \boxed{-0.9435}$$

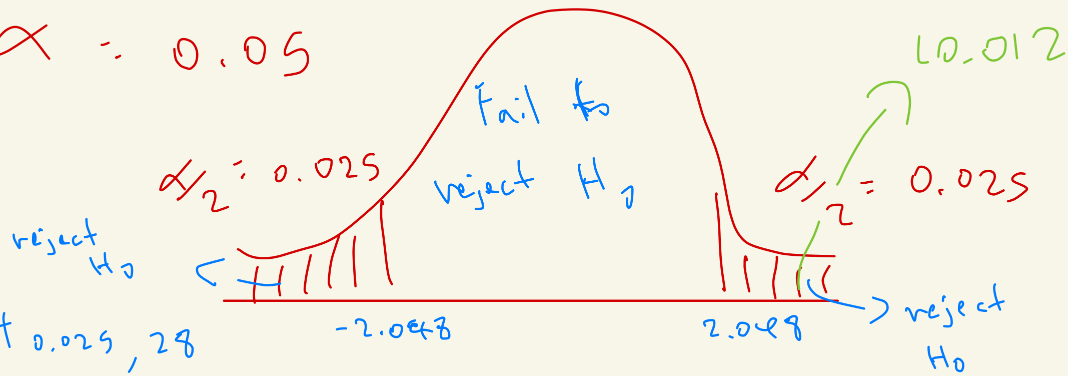
$|t_{cal}| < |t_{table}|$ $(0.9435 < 2.048)$ fall

in fall in reject H_0 region. I believe coefficients are not different from 0 at $\alpha = 0.05$

$H_0 : \beta_1 = 0$

$H_1 : \beta \neq 0$

$\alpha = 0.05$



$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{S_e \hat{\beta}_1} = \frac{22.9698 - 0}{\sqrt{5.2635}}$$

$$= 10.012$$

$|t_{call}| > |t_{table}|$ $(10.012 > 2.028)$

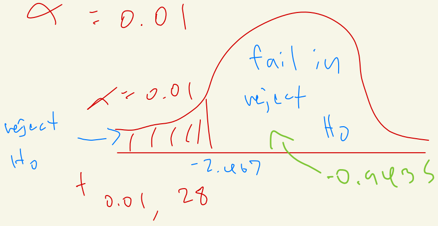
fall in reject H_0. I believe coefficient are different from 0 at $\alpha = 0.05$

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

$H_0 : \beta_2 \geq 0$

$H_1 : \beta_2 < 0$

$\alpha = 0.01$



$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{S_e \hat{\beta}_2} = \frac{-0.159 - 0}{\sqrt{0.0284}}$$

$$= -0.9435$$

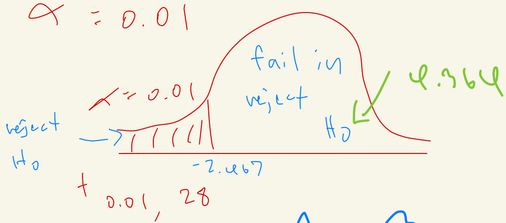
$t_{cal} > t_{table}$ $(-0.9435 > -2.467)$

fall in fail to reject H_0. I believe coefficient not less than 0 at $\alpha = 0.01$

$H_0 : \beta_1 \geq 0$

$H_1 : \beta_1 < 0$

$\alpha = 0.01$



$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{S_e \hat{\beta}_1} = \frac{22.9698 - 0}{\sqrt{5.2635}}$$

$$= 10.012$$

$t_{cal} > t_{table}$ $(0.012 > -2.467)$

fall in fail to reject H_0. I believe coefficient not less than 0 at $\alpha = 0.01$

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = \overset{\hat{\beta}_1}{7{,}836} - \overset{\hat{\beta}_2}{502.4}X_i$$

$$\underset{se\,\hat{\beta}_1}{(52)} \qquad \underset{se\,\hat{\beta}_2}{(411.8)}$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45,$

$\hat{\sigma}^2 = 212{,}877,$

$\sum(X_i - \bar{X})^2 = 78.73,$

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

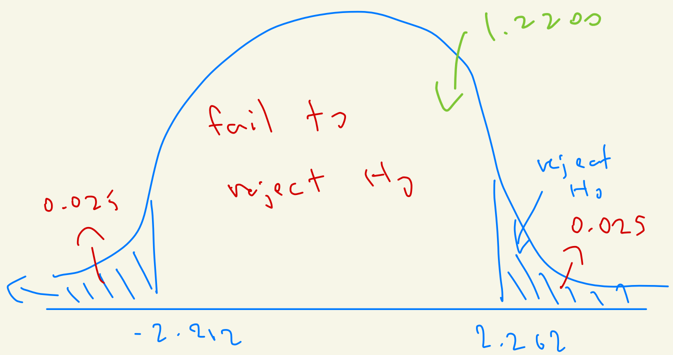
d) Calculate the elasticity of market price when a car is 10 years old.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

$H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

$\alpha = 0.05$



$t_{9, 0.025} = 2.262$

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{Se\,\hat{\beta}_2} = \frac{-502.4 - 0}{411.8}$$

$= \boxed{1.2200}$

$|t_{cal}| < |t_{cri}| : 1.2200 < 2.262$

fail to reject H_0 region, we believe $\hat{\beta}_2 = 0$ which means x no effect to y at 95% confidence level

Ans: Not make sense

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

- $\boxed{\hat{Y}_0 - (t_{\alpha/2} \cdot \hat{\sigma}_{\hat{Y}_0}) \leq Y_0 \leq \hat{Y}_0 + (t_{\alpha/2} \cdot \hat{\sigma}_{\hat{Y}_0})}$

- $\hat{\sigma}^2 \left(\frac{1}{n} + \frac{(x_0 - \bar{x})^2}{\Sigma(x_i - \bar{x})^2} \right) = 212{,}877 \left[\frac{1}{11} + \frac{(5 - 7.45)^2}{78.73} \right]$

$= 35{,}582.5345$

- $x_i = 5 \longrightarrow \hat{Y}_0 = 7{,}836 - 502.4(5) = 5324$

- $\hat{\sigma}_{\hat{Y}_0} = \sqrt{35{,}582.5345} = 188.6333$

- $t_{\frac{0.05}{2}} = t_{0.025, 9} = 2.262$

- $5{,}324 - (2.262 \cdot 188.6333) \leq Y_0 \leq 5{,}324 + (2.262 \cdot 188.6333)$

$$4{,}897.3115 \leq Y_0 \leq 5750.6885$$

∴ If his car will be averagely priced at when his car is 5 years old, we estimate price in $(4{,}897.3115, 5750.6885)$ at 95% confidence level.

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

Se — $\pm$ not change

Se — $\times, \div$ change

Due to the given, when multiply all the X with 10, the new SRF with the standard error would be change by multiply by 10 to all variable of X.

d) Calculate the elasticity of market price when a car is 10 years old.

$$e_p = \frac{dQ/Q}{dP/P} = \frac{dQ}{dP} \times \frac{P}{Q}$$

$$A: \frac{dP}{dA} \times \frac{A}{P} = -5,024 \times \frac{10}{2812} = -1.7866$$

$\therefore |-1.7866| = 1.7866 > 1$, so it is elasticity