FN452: Asset management and portfolio analysis

Index models, CAPM, and multifactor models

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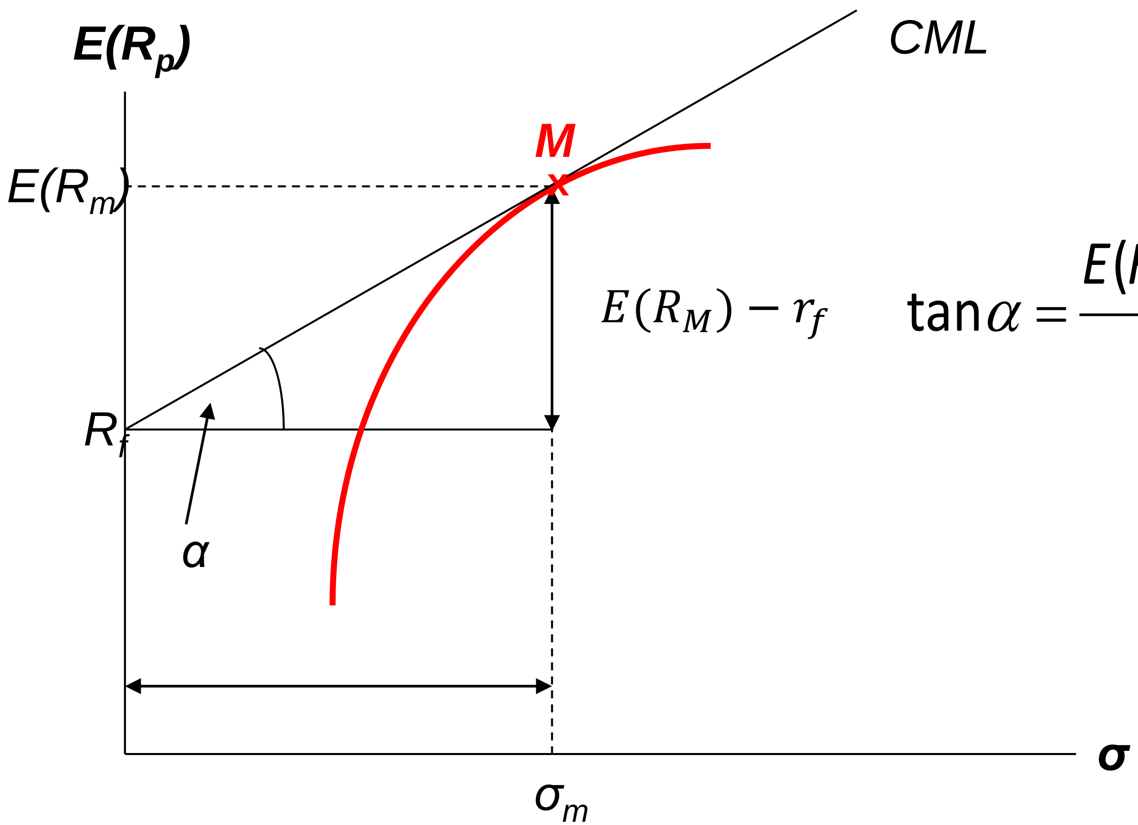
- The Capital Market Line (CML)
- Capital Asset Pricing Model (CAPM) and The Security Market Line (SML)
- The CAPM and the index model
- Multifactor models

The Capital Market Line (CML)

- The Capital Market Line (CML) is the capital allocation line in a world in which all investors agree on the expected returns, standard deviations, and correlations of all assets (aka the "homogeneous expectations" assumption)
- Assuming identical expectations, there will be only one capital allocation line, and it is called the CML

The Capital Market Line (CML)



$$\tan\alpha = \frac{E(R_M) - R_f}{\sigma_M}$$

M is called the tangency portfolio

The Capital Market Line (CML)

- Optimal risky portfolio (the tangency portfolio) is the market portfolio (M) which includes every risky asset
- Assumes homogenous expectations: All investors agree on expected returns, variances, and covariances

$$E(R_c) = R_f + \left[\frac{E(R_M) - R_f}{\sigma_M} \right] \sigma_c$$

- The CAL assumes heterogeneous expectations
- The CAL is different for investors with different expectations. Investors use different efficient frontiers and tangency portfolios

$$E(R_c) = R_f + \left[\frac{E(R_P) - R_f}{\sigma_P} \right] \sigma_c$$

The Capital Asset Pricing Model (CAPM)

Assumptions

- Markets are competitive, equally profitable
  - No investor is wealthy enough to individually affect prices
  - All information publicly available
  - No taxes on returns, no transaction costs
  - Unlimited borrowing/lending at risk-free rate
- Investors are alike **except for initial wealth, risk aversion**
  - Investors plan for single-period horizon; they are rational, mean-variance optimizers
  - Use same inputs, consider identical portfolio opportunity sets

The Capital Asset Pricing Model (CAPM)

- Implications from the assumptions

1) All investors choose to hold market portfolio

2) Market portfolio is on efficient frontier, and it will be the optimal risky portfolio

3) Risk premium on market portfolio is equal to the product of variance of market portfolio and average investor's risk aversion

$$E(R_M) - r_f = \bar{A}\sigma_M^2$$

4) Risk premium on individual assets can be explained as

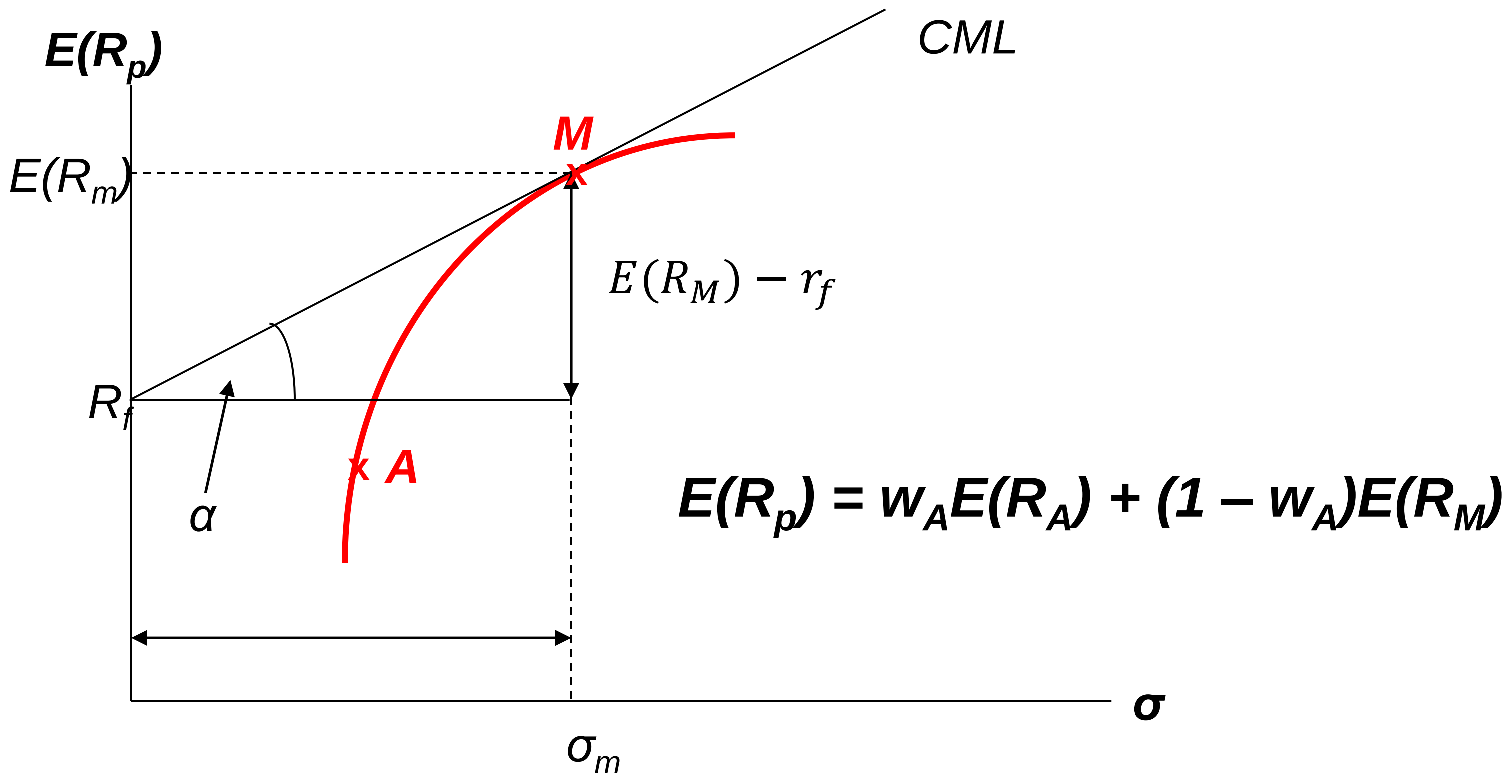
$$E(R_i) - R_f = \beta\left[E(R_M) - R_f\right]$$

The Capital Asset Pricing Model (CAPM)

Why all investors would hold the market portfolio?

- According to previous assumptions, investors must arrive at the same determination of the optimal risk portfolio
- Same proportion of stocks for each investor
- If excluding one stock → that stock price will fall sharply → investors will buy it → ultimately, the stock will reach an optimal price and be included in the market portfolio
- This price adjustment process guarantees that all stocks will be included in the optimal portfolio

The Capital Asset Pricing Model (CAPM)



M is called the tangency portfolio

The Capital Asset Pricing Model (CAPM)

- The expected return from a marginal investment in the inefficient portfolio is

$$\frac{dE(R_p)}{dw_A} = E(R_A) - E(R_M)$$

- The marginal risk produced by a marginal investment in *A* is

$$\sigma_p \quad = \sqrt{\left[w_A^2\sigma_A^2 + (1-w_A)^2\sigma_M^2 + 2w_A(1-w_A)Cov(R_A,R_M)\right]}$$

$$\frac{d\sigma_p}{dw_A} = \frac{1}{2}\left[w_A^2\sigma_A^2 + (1-w_A)^2\sigma_M^2 + 2w_A(1-w_A)Cov(R_A,R_M)\right]^{-\frac{1}{2}}$$

$$\bullet\left[2w_A\sigma_A^2 - 2(1-w_A)\sigma_M^2 + 2Cov(R_A,R_M) - 4w_ACov(R_A,R_M)\right]$$

The Capital Asset Pricing Model (CAPM)

- At point M, $w_A = 0$, we obtain

$$\frac{d\sigma_p}{dw_A} = \frac{1}{2}\left[\sigma_M^2\right]^{-\frac{1}{2}}\left[2Cov(R_A, R_M) - 2\sigma_M^2\right]$$

$$\frac{d\sigma_p}{dw_A} = \frac{Cov(R_A, R_M) - \sigma_M^2}{\left(\sigma_M^2\right)^{\frac{1}{2}}} = \frac{Cov(R_A, R_M) - \sigma_M^2}{\sigma_M}$$

- Tangent point M on the frontier AM is

$$\frac{\frac{dE(R_p)}{dw_A}}{\frac{d\sigma_p}{dw_A}} = \frac{dE(R_p)}{d\sigma_p} = \frac{E(R_A) - E(R_M)}{\frac{\left(Cov(R_A, R_M) - \sigma_M^2\right)}{\sigma_M}} \Rightarrow \frac{(E(R_A) - E(R_M))\sigma_M}{Cov(R_A, R_M) - \sigma_M^2}$$

The Capital Asset Pricing Model (CAPM)

- The CML is also tangent to point *M*, so

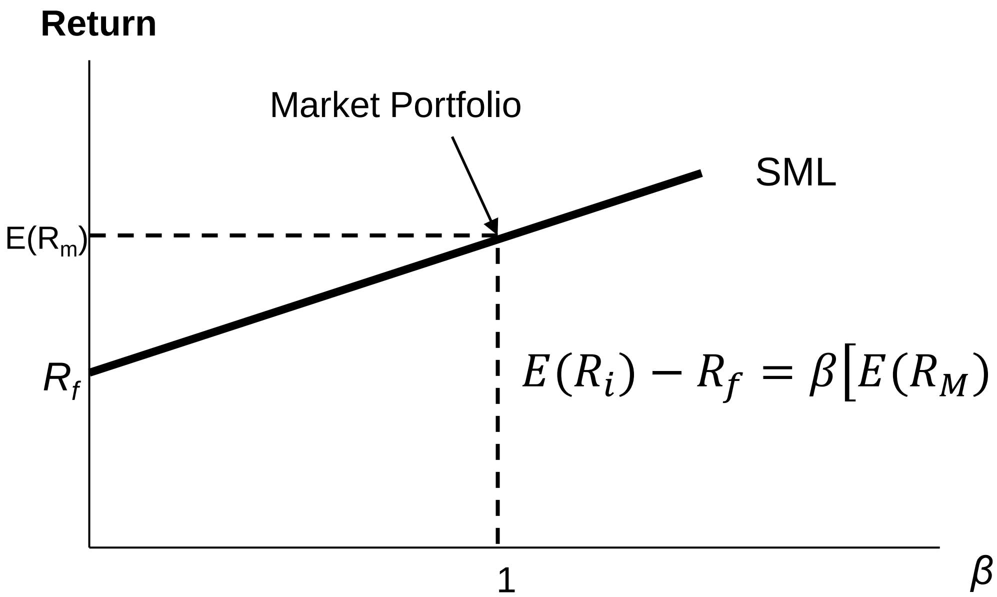
$$\frac{[E(R_A) - E(R_M)]\sigma_M}{Cov(R_A, R_M) - \sigma_M^2} = \frac{E(R_M) - R_f}{\sigma_M}$$

$$[E(R_A) - E(R_M)] = \frac{\left[E(R_M) - R_f\right]\left[Cov(R_A, R_M) - \sigma_M^2\right]}{\sigma_M^2}$$

$$[E(R_A) - E(R_M)] = \beta\left[E(R_M) - R_f\right] - \left[E(R_M) - R_f\right], \beta = \frac{Cov[R_A, R_M]}{\sigma_M^2}$$

$$E(R_A) - R_f = \beta\left[E(R_M) - R_f\right]$$

The Capital Asset Pricing Model (CAPM)



$$E(R_i) - R_f = \beta\left[E(R_M) - R_f\right], \beta = \frac{Cov[R_i, R_M]}{\sigma_M^2}$$

- SML is a graph of the Capital Asset Pricing Model (CAPM)
  - Intercept is the risk-free rate, R_f
  - Slope is market risk premium, $E(R_m) - R_f$
  - Beta: Measure of systematic risk

The Capital Asset Pricing Model (CAPM)

- If the expected return on the market rises by 10%
  - $E(R_i)$ will rise > 10% if β > 1
  - $E(R_i)$ will rise < 10% if β < 1
  - $E(R_i)$ will rise = 10% if β = 1
  - Higher beta stocks will outperform the market in a bull run and lower beta shares will under-perform
  - Conversely high beta stocks will fall faster in a bear market

The Capital Asset Pricing Model (CAPM)

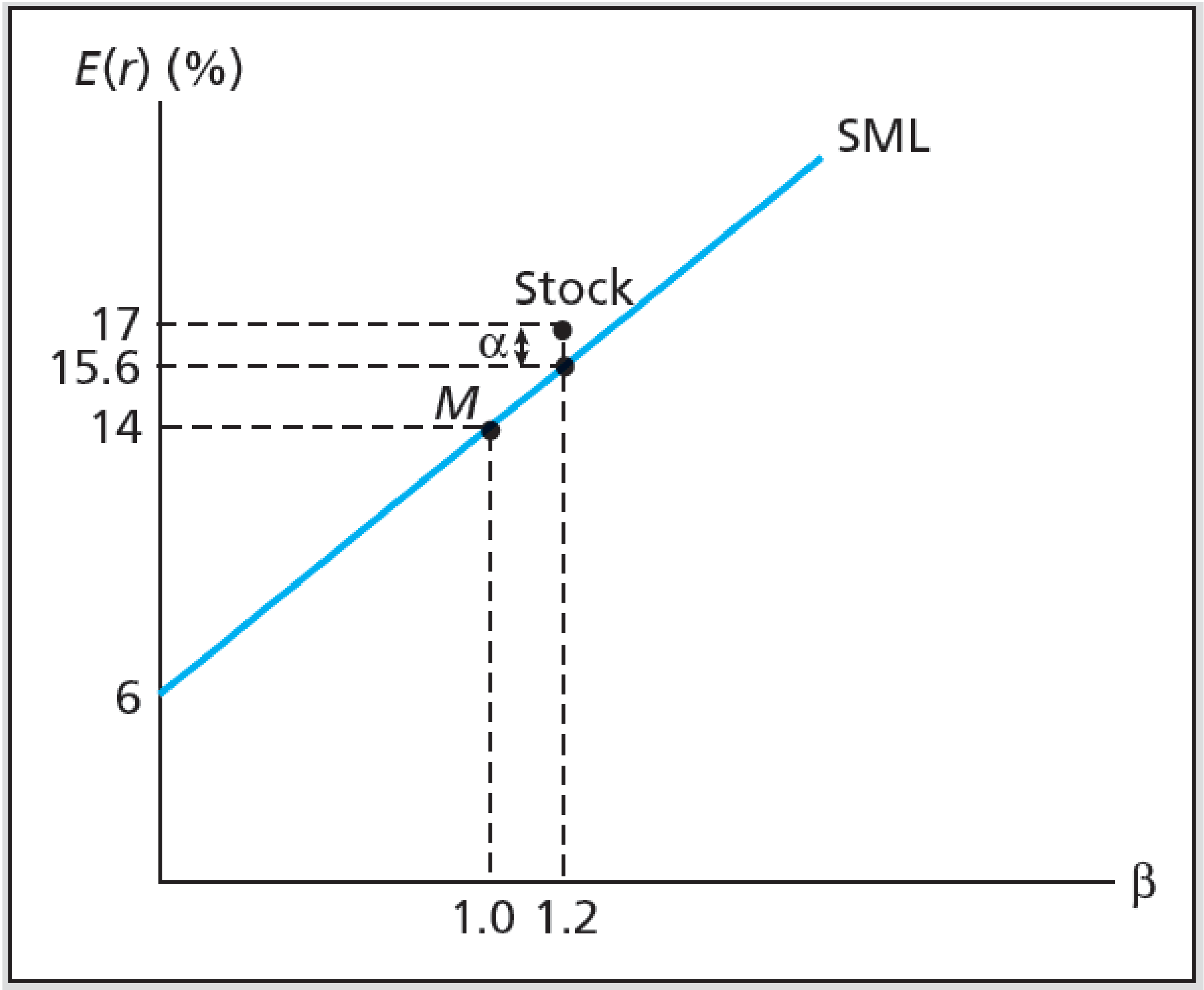
The Security Market Line (SML)

- Represents expected return-beta relationship of CAPM
- Graphs individual asset risk premiums as function of asset risk
- Whenever the CAPM holds, all securities must lie on the SML → "Fairly priced" assets plot exactly on the SML

Alpha

- Abnormal rate of return on security in excess of that predicted by equilibrium model (CAPM)
- Underpriced assets plot above the SML

The Capital Asset Pricing Model (CAPM)



Source: Bodie, Kane, and Marcus (2021)

CAPM and index models

- Index Model, Realized Returns, Mean-Beta Equation
  - $r_{it} - r_{ft} = \alpha_i + \beta_i(r_{Mt} - r_{ft}) + e_{it}$
    - r_{it}: HPR
      - *i*: Asset
      - *t*: Period
    - α_i: Intercept of security characteristic line
    - β_i: Slope of security characteristic line
    - r_M: Index return
    - e_{it}: Firm-specific effects
  - $E(r_{it}) - r_{ft} = \alpha_i + \beta_i[E(r_{Mt}) - r_{ft}]$

Source: Bodie, Kane, and Marcus (2021)

CAPM and index models

- Estimating Index Model
  - $R_{Gt} = \alpha_G + \beta_G R_{Mt} + e_{Gt}$
    - $R_G = r_G - r_f$, excess return
    - Residual = Actual return – Predicted return for Google
    - $e_{Gt} = R_{Gt} - (\alpha_G + \beta_G R_{Mt})$

Source: Bodie, Kane, and Marcus (2021)

CAPM and index models

TABLE 7.2 Security characteristic line for Google (S&P 500 used as market index), January 2006–December 2010

Linear Regression

Regression Statistics	(This table produced by StatPlus patch for Mac Excel, which lacks the Data Analysis tool of Windows Excel)
R (correlation)	0.5914
R-square	0.3497
Adjusted *R*-square	0.3385
SE of regression	8.4585
Total number of observations	60

Regression equation: Google (excess return) = 0.8751 + 1.2031 * S&P 500 (excess return)

ANOVA

	df	*SS*	*MS*	*F*	*p*-level
Regression	1	2231.50	2231.50	31.19	0.0000
Residual	58	4149.65	71.55		
Total	59	6381.15			

	Coefficients	**Standard Error**	***t*-Statistic**	***p*-value**	**LCL**	**UCL**
Intercept	0.8751	1.0920	0.8013	0.4262	−1.7375	3.4877
S&P 500	1.2031	0.2154	5.5848	0.0000	0.6877	1.7185

t-Statistic (2%)	2.3924

LCL—Lower confidence interval (95%)
UCL—Upper confidence interval (95%)

Source: Bodie, Kane, and Marcus (2021)

CAPM and index models

Table 7.2 summary

- **Correlation** of Google with the S&P 500 is 0.5914.
- The model explains about 34% of the variation in Google.
- Google's **Alpha** is 0.87% per month (10.50% annually) but it is not statistically significant
- Google's **Beta** is 1.2031, but the 95% confidence interval is 0.69 to 1.72

CAPM and index models

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Statistical inferences

- Positive Alpha of 0.88 means that Google stock was above the SML for this period
- But this estimate has standard error of 1.09 → leading to t-statistic of 0.80, indicating low statistical significance

$$t = \frac{\text{estimated value} - \text{hypothesis value}}{\text{standard error of estimate}}$$

- This is reflected in the large p-value of 0.43 → probability that an estimate of Alpha could have resulted from pure chance
- Confidence interval – with 95% confidence, the true Alpha lies in the wide interval from -1.74 to 3.49 → **cannot conclude that Google's true Alpha is not zero**

CAPM and index models

Statistical inferences

- The estimate Beta is 1.20 with SE of 0.21 → resulting in a t-statistic of 5.58
- And practically zero p-value for the hypothesis that the true Beta is in fact zero (or probability of observing an estimate this large if the true Beta were zero is negligible)
- We can also test if Beta is significantly different from 1

$$t = \frac{\text{estimated value} - \text{hypothesis value}}{\text{standard error of estimate}} = \frac{1.2031 - 1}{0.2154} = 0.94$$

- This value is below the conventional threshold for statistical significance; we cannot say with confidence that Google's Beta differs from 1

CAPM and the real world

- Portfolio theory and the CAPM have become accepted tools in the practitioner community
- Early tests in 1970s were partially supportive of the CAPM
    - Average returns were higher for higher-beta portfolios, but…
    - The reward for beta risk was less than predicted by the CAPM
- Also, **Roll (1977)** argued that since the true market portfolio can never be observed, the CAPM is necessarily **untestable**

CAPM and the real world

- **Fama and French (1992)** claimed that in contradiction of the CAPM, certain characteristics of the firm, namely, **size and the ratio of market to book value**, were far more useful in predicting future returns than Beta
- Other tests also confirm that Beta does not tell the whole story of risk → Other risk factors that affect security returns beyond Beta
- However, more sophisticated models of security pricing all rely on the key distinction between systematic and diversifiable risk → The CAPM provides a useful framework

Multifactor models

- Multifactor models
  - Models of security returns that respond to several systematic factors
  - Two-index portfolio in realized returns
    - $R_{it} = \alpha_i + \beta_{iM} R_{Mt} + \beta_{iTB} R_{TBt} + e_{it}$
  - Two-factor SML
    - $E(r_i) = r_f + \beta_{iM}\left[E(r_M) - r_f\right] + \beta_{iTB}\left[E(r_{TB}) - r_f\right]$

Source: Bodie, Kane, and Marcus (2021)

Multifactor models – Fama-French 3-Factor Model

- **Fama and French (1992)** claimed that in contradiction of the CAPM, certain characteristics of the firm, namely, **size and the ratio of market to book value**, were far more useful in predicting future returns than Beta
- Other tests also confirm that Beta does not tell the whole story of risk → Other risk factors that affect security returns beyond Beta
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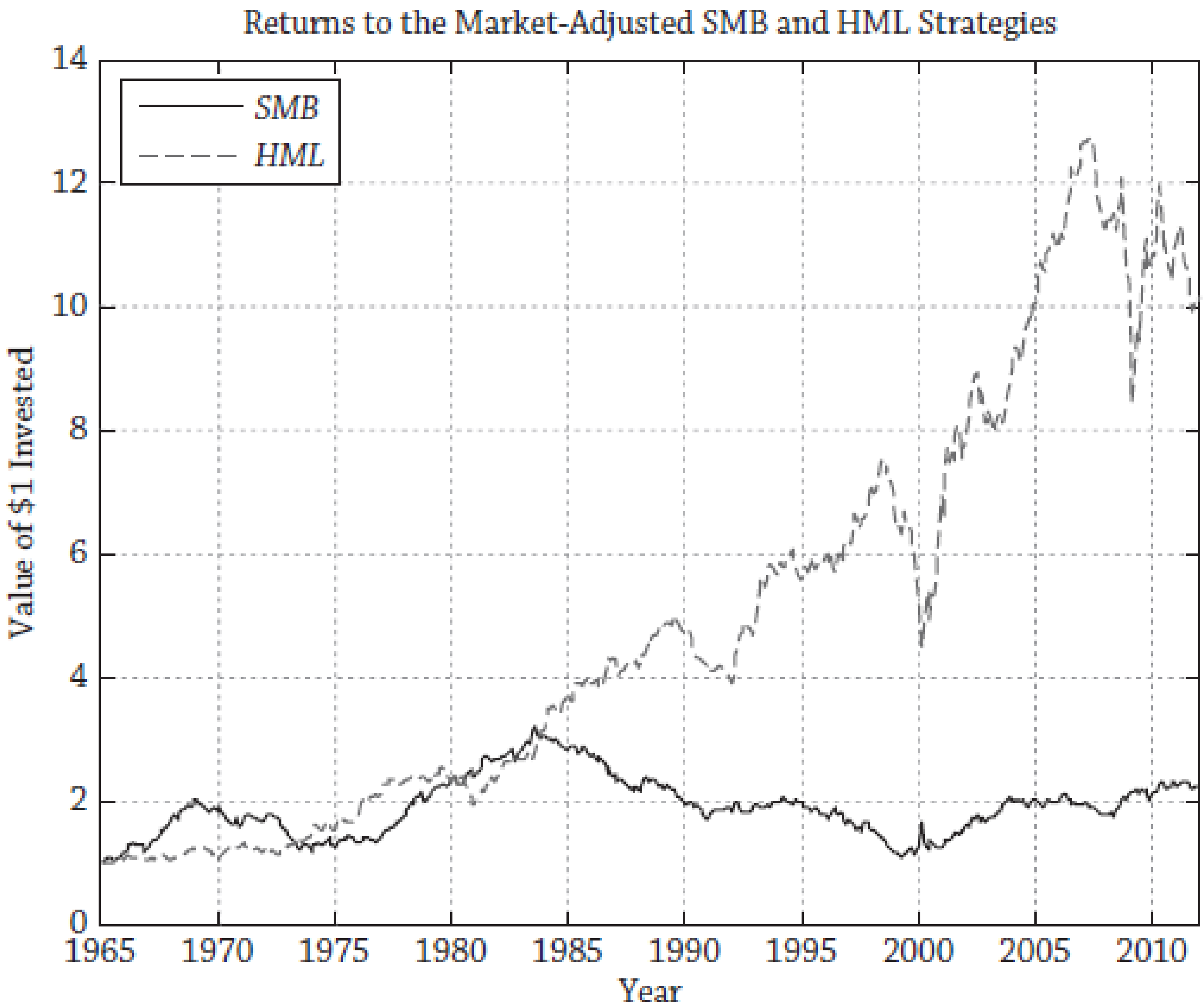
Multifactor models

- Fama-French Three-Factor Model
  - $r_G - r_f = \alpha_G + \beta_M(r_M - r_f) + \beta_{HML} r_{HML} + \beta_{SMB} r_{SMB} + e_G$
  - Estimation results
    - Three aspects of successful specification
      - Higher adjusted R-square
      - Lower residual SD
      - Smaller value of alpha

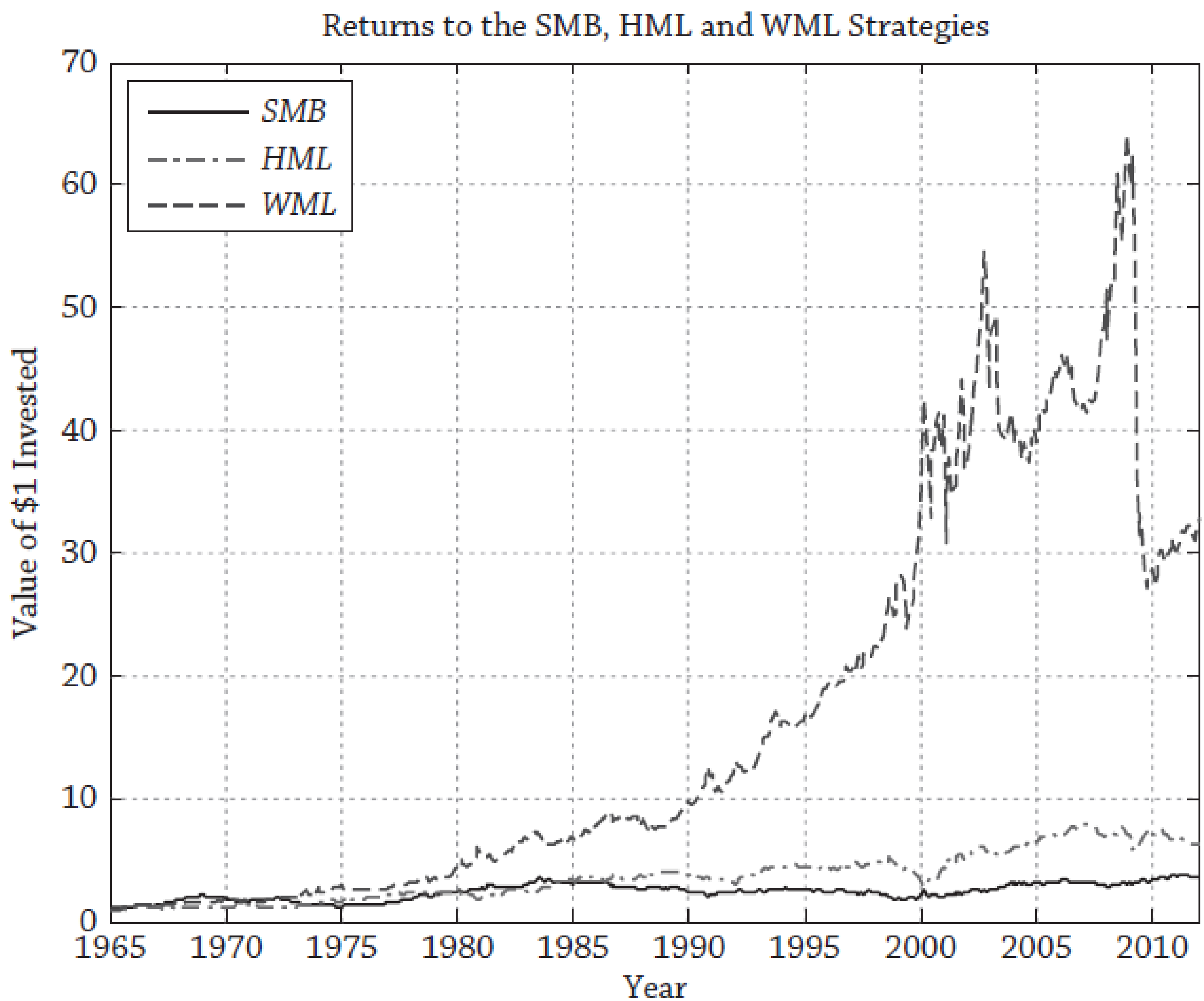
Source: Bodie, Kane, and Marcus (2021)

Multifactor models – Why size and value?



Source: Ang (2014) Figure 7.6

Multifactor models – Why momentum?



Source: Ang (2014) Figure 7.7

Multifactor models – An example

- Consider the CAPM applied to an asset with a beta of 1.3, say:

$$E(R_p) - R_f = \beta_p\left[E(R_m) - R_f\right]$$
$$E(R_p) = R_f + 1.3\left[E(R_m) - R_f\right]$$
$$E(R_p) = -0.3R_f + 1.3[E(R_m)]$$

- Thus, \$1 in asset *i* is equivalent to a short position of \$0.30 in cash (T-bills) and a leveraged position of \$1.30 in the market portfolio. This is a replicating portfolio.
- The **beta** is actually a mimicking **portfolio weight**
- If there is alpha the fund manager is generating that in excess of the \$0.30 in cash and the \$1.30 in the market portfolio

Multifactor models – An example

- Example: Berkshire Hathaway, monthly returns 1996:06-2018:01, using Fama & French (1993) Benchmark

	Coefficients	*Standard Error*	*t Stat*
Intercept	0.4238	0.2758	1.5365
RMRF	0.6365	0.0631	10.0937
SMB	-0.4634	0.0846	-5.4809
HML	0.4747	0.0896	5.2991

- $1 invested in BRK is economically equivalent to

  (1-0.64) = 0.36 in T-bills

  0.64 in the market portfolio

  -0.46 in SMB = -0.46 in small caps and 0.46 in large caps

  0.44 in HML = 0.47 in value stocks and -0.47 in growth stocks

  plus 0.42% (alpha) in returns per month

Multifactor models – An example

- Fama-French-Carhart 4-factor model
- The fourth factor is momentum
  - Buying stocks that have gone up over the past year (winners) and shorting stocks with the lowest returns over the same period (losers)

	Coefficients	*Standard Error*	*t Stat*
Intercept	0.4160	0.2794	1.4889
RMRF	0.6408	0.0670	9.5579
SMB	-0.4650	0.0851	-5.4642
HML	0.4788	0.0923	5.1889
MOM	0.0108	0.0565	0.1916