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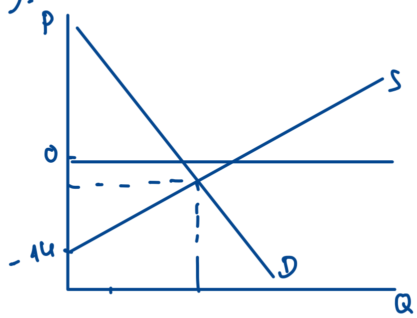
EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If "a" increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

1.1).



1.2). Market demand : $P_d = 10 - Q^2$

Market supply : $Q = a - P \Rightarrow P_s = a - Q$; $a = -14$

At equilibrium: $P_d = P_s$

$10 - Q^2 = -14 - Q$

$Q^2 - Q - 24 = 0$

$Q^* = 5.424$

$Q = -4.424$ X

$\Rightarrow P^* = -14 - 5.424 = -19.424$

1.3). When a increase to -12, supply curve will shift up, result in increase P and decrease Q.

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

Solution: $R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right)$

$$R'(Q) = \frac{2Q}{Q^2+1} + 3\left(\frac{Q+1-Q}{(Q+1)^2}\right)$$

$$= \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2} > 0 \; ; \forall Q \geq 0$$

Since $Q \geq 0 \Rightarrow R(Q) > 0$

The marginal revenue function is an increasing function.

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

Solution:

$$\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$$

$$\Rightarrow \pi'(Q) = -\frac{3}{3}Q^2 - 2Q + 8$$

$$= -Q^2 - 2Q + 8$$

let: $\pi'(Q) = 0 \Rightarrow -Q^2 - 2Q + 8 = 0$

$$(-Q+2)(Q+4) = 0$$

$$Q = -4, 2$$

$-9+6+8$

Q		-4		2	
$\pi'(Q)$	$-$	0	$+$	0	$-$

$$\pi''(Q) = -2Q - 2$$

$$\pi''(-4) = -2(-4) - 2$$

$$= 6 > 0$$

$$\pi''(2) = -2(2) - 2$$

$$= -2 < 0 \Rightarrow \text{Max.}$$

$$\pi(2) = -\frac{1}{3}(2)^3 - 2^2 + 8(2) - 1 = -\frac{8}{3} + 11 = \frac{25}{3}$$

$$\boxed{\text{The level of profit-maximizing output} : \pi(2) = \frac{25}{3}}$$

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B

Cannot compute because it is not the same size.

4.2 A*B

$$A \times B = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 8+36 & 16+45 & 24+54 \\ 10+44 & 20+55 & 30+66 \end{bmatrix} = \begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$$

4.3 det(A)

$$\det(A) = 8 \times 11 - 10 \times 9 = -2$$

4.4 det(B)

Cannot be compute because det can computed only for an n by n square matrix.

4.5 det(C)

$$\det(C) = 1(5 \times 9 - 8 \times 6) - 2(4 \times 9 - 7 \times 6) + 3(4 \times 8 - 5 \times 7)$$

$$= -3 + 12 - 9 = 0$$

5. Suppose that $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

Solution: $U(x,y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$

$$\frac{\partial U}{\partial x} = \frac{\partial}{\partial x}\left(x^a y^b + \ln(x) - \ln(x+y)\right)$$

$$= a y^b x^{a-1} + \frac{1}{x} - \frac{1}{x+y}$$

$$\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left(x^a y^b + \ln(x) - \ln(x+y)\right)$$

$$= b x^a y^{b-1} - \frac{1}{x+y}$$