EE320 – Comparative Static Analysis

Tax Incidence and Price Elasticities

Given the demand and supply function:

$$Q_d = a - bP \qquad \text{and} \qquad Q_s = -c + dP$$

where a, b, c, and d are positive constants.

One can show that the equilibrium price and quantities for the above market are:

$$P^* = \frac{a+c}{b+d} \qquad \text{and} \qquad Q^* = \frac{ad-bc}{b+d}.$$

If a specific tax T per unit is imposed either on the consumer or the producer, *the after-tax equilibrium price for the consumer* can be derived as:

$$P_d^* = \frac{a+c+dT}{b+d},$$

and the *after-tax equilibrium prices for the producer* can be derived as:

$$P_s^* = \frac{a+c-dT}{b+d}\ .$$

Accordingly, the per-unit tax burden on consumer and the per-unit tax burden on producer will be $\frac{dT}{b+d}$ and $\frac{bT}{b+d}$, respectively.

Question: Suppose that that the amount of tax T changes, what would happen to the after-tax equilibrium prices for consumer and producer?

i.e. $\frac{\Delta P_d^*}{\Delta T} =?$ and $\frac{\Delta P_s^*}{\Delta T} =?$

Let's suppose that the specific tax increases from $T_0$to T_1. So, $\Delta T = T_1 - T_0$.

Case 1: The impact of a (specific) tax change on the consumer's price.

Recall: $P_d^* = \frac{a+c+dT}{b+d}$

$$\Delta P_d^* = P_d^{1*} - P_d^{0*} = \frac{a+c+dT_1}{b+d} - \frac{a+c+dT_0}{b+d} = \frac{d(T_1-T_0)}{b+d} = \frac{d}{b+d} \cdot \Delta T$$

Hence, $\frac{\Delta P_d^*}{\Delta T} = \frac{d}{b+d}$. -- (1)

Next, we want to express the expression in equation (1) in terms of the price elasticity of demand and price elasticity of supply.

Let the notations for the price elasticities be given by:

(i) Price elasticity of demand (in absolute term) :

$$\eta = \frac{-\Delta Q_d / Q_d}{\Delta P / P} = \frac{-\Delta Q_d}{\Delta P} \cdot \frac{P}{Q_d} \qquad (3)$$

(ii) Price elasticity of supply:

$$\varepsilon = \frac{\Delta Q_s / Q_s}{\Delta P / P} = \frac{\Delta Q_s}{\Delta P} \cdot \frac{P}{Q_s} \qquad (4).$$

Moreover, from the demand and supply equations given above, we can write the impact of a price change on the quantity demanded and on the quantity supplied as follows:

$\Delta Q_d = Q_{d1} - Q_{d0} = a - b(P_1 - P_0) = a - b \cdot \Delta P \rightarrow \frac{\Delta Q_d}{\Delta P} = -b$ or $-\frac{\Delta Q_d}{\Delta P} = b$;

$\Delta Q_s = Q_{s1} - Q_{s0} = -c + d(P_1 - P_0) = -c + d \cdot \Delta P \rightarrow \frac{\Delta Q_s}{\Delta P} = d$;

By substituting $\frac{\Delta Q_d}{\Delta P}$ and $\frac{\Delta Q_s}{\Delta P}$ into equation (1), we have:

$$\frac{\Delta P_d^*}{\Delta T} = \frac{d}{b+d} = \frac{\frac{\Delta Q_S}{\Delta P}}{-\frac{\Delta Q_d}{\Delta P}+\frac{\Delta Q_S}{\Delta P}}.$$

Multiply both the numerator and denominator on RHS by $\frac{P}{Q}$, and we get:

$$\frac{\Delta P_d^*}{\Delta T} = \frac{\left(\frac{\Delta Q_S}{\Delta P}\right)\times\frac{P}{Q}}{\left(-\frac{\Delta Q_d}{\Delta P}\right)\times\frac{P}{Q}+\left(\frac{\Delta Q_S}{\Delta P}\right)\times\frac{P}{Q}} \qquad \text{-- (5)}$$

From (3) and (4), $\eta = \frac{-\Delta Q_d}{\Delta P}\cdot\frac{P}{Q_d}$ and $\varepsilon = \frac{\Delta Q_S}{\Delta P}\cdot\frac{P}{Q_S}$. Substitute both η and ε in (5).

Hence, one can show that:

$$\frac{\Delta P_d^*}{\Delta T} = \frac{\varepsilon}{\eta+\varepsilon}. \qquad \text{-- (6)}$$

⇨ If $\varepsilon = 0$, then $\frac{\Delta P_d^*}{\Delta T} = 0$.

(i.e. If the supply is perfectly price inelastic, there will be no change in the consumer's after-tax equilibrium price as a result of a specific-tax change.)

⇨ If $\eta = 0$, then $\frac{\Delta P_d^*}{\Delta T} = 1$.

(i.e. If the demand is perfectly price inelastic, the consumer's after-tax equilibrium price will change by the amount of the change in the specific tax.)

Case 2: The impact of a (specific) tax change on the producer's price.

Recall: $P_S^* = \frac{a+c-bT}{b+d}$

$$\Delta P_S^* = P_S^{1*} - P_S^{0*} = \frac{a+c-bT_1}{b+d} - \frac{a+c-bT_0}{b+d} = \frac{-b(T_1-T_0)}{b+d} = \frac{-b}{b+d} \cdot \Delta T$$

Hence, $\frac{\Delta P_S^*}{\Delta T} = \frac{-b}{b+d}$. -- (7)

Next, we want to express the expression in equation (7) in terms of the price elasticity of demand and price elasticity of supply.

Recall from the above, $\frac{\Delta Q_d}{\Delta P} = -b$ and $\frac{\Delta Q_s}{\Delta P} = d$. Substitute these two terms in (7), and we have:

$$\frac{\Delta P_S^*}{\Delta T} = \frac{-b}{b+d} = \frac{\frac{\Delta Q_d}{\Delta P}}{-\frac{\Delta Q_d}{\Delta P} + \frac{\Delta Q_S}{\Delta P}}.$$

Multiply both the numerator and denominator on RHS by $\frac{P}{Q}$, and we get:

$$\frac{\Delta P_S^*}{\Delta T} = \frac{\left(\frac{\Delta Q_d}{\Delta P}\right) \times \frac{P}{Q}}{\left(-\frac{\Delta Q_d}{\Delta P}\right) \times \frac{P}{Q} + \left(\frac{\Delta Q_S}{\Delta P}\right) \times \frac{P}{Q}} \quad \text{-- (8).}$$

Since $\eta = \frac{-\Delta Q_d}{\Delta P} \cdot \frac{P}{Q_d}$ and $\varepsilon = \frac{\Delta Q_s}{\Delta P} \cdot \frac{P}{Q_s}$, we have:

$$\frac{\Delta P_S^*}{\Delta T} = \frac{-\eta}{\eta + \varepsilon} \quad \text{-- (9).}$$

⇨ If $\varepsilon = 0$, then $\frac{\Delta P_S^*}{\Delta T} = -1$.

(i.e. If the supply is perfectly price inelastic, the producer's after-tax equilibrium price will decrease by the amount of the change in the specific tax.)

⇨ If $\eta = 0$, then $\frac{\Delta P_s^*}{\Delta T} = 0$.

(i.e. If the demand is perfectly price inelastic, there will be no change in the producer's after-tax equilibrium price as a result of a specific-tax change.)

A Numerical Example

Suppose that the price elasticity of demand (in absolute term) and the price elasticity of supply are 0.2 and 0.84, respectively. Suppose also that a specific tax of $2 per unit is currently imposed on the consumer.

Question: If the tax rate changes to $4 per unit, how much does the consumer's after-tax equilibrium price change? What about the impact on the producer's after-tax equilibrium price?

Answer:

From above derivation, we have:

$$\frac{\Delta P_d^*}{\Delta T} = \frac{\varepsilon}{\eta+\varepsilon} = \frac{0.84}{0.2+0.84} = 0.8$$

Hence, if $\Delta T = \$4 - \$2 = \$2$, then $\Delta P_d^* = 0.8 * \$2 = \$1.6$.

Similarly, we have shown that:

$$\frac{\Delta P_s^*}{\Delta T} = \frac{-\eta}{\eta+\varepsilon} = \frac{-0.2}{0.2+0.84} = -0.19$$

Hence, if $\Delta T = \$2$, then $\Delta P_s^* = -0.19 * \$2 = -\$0.38$.