Assignment 5: Optional Problems

1. Define $f : \mathbb{Z} \to \mathbb{Z}$ by the rule $f(n) = 2 - 3n$, for all integers n.

   (i) Is f one-to-one? Prove or give a counterexample.

   (ii) Is f onto? Prove or give a counterexample.

2. Define $G : \mathbb{R} \to \mathbb{R}$ by the rule $G(x) = 2 - 3x$ for all real numbers x. Is G onto? Prove or give a counterexample.

3. Define $f : \mathbb{R} - \{0\} \to \mathbb{R}$ by $f(x) = \frac{x+1}{x}$, for all real numbers $x \neq 0$. Determine whether or not f is one-to-one and justify your answer.

4. Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = \frac{x}{x^2+1}$, for all real numbers x. Determine whether or not f is one-to-one and justify your answer.

5. Define $G : \mathbb{R} \times \mathbb{R} \to \mathbb{R} \times \mathbb{R}$ as follows: $G(x, y) = (2y, -x)$ for all $(x, y) \in \mathbb{R} \times \mathbb{R}$.

   a. Is G one-to-one? Prove or give a counterexample.

   b. Is G onto? Prove or give a counterexample.