Risk Preferences

PROSPECT THEORY II: VALUE FUNCTION

EE416 SEM1/2020

Recap: Optimal sin tax

Actual $x^*(t)$ satisfies $v_x(x^*(t);\rho) - \beta c_x(x^*(t);\gamma) - (1+t) = 0$

$$v_x(x^*;\rho) - \beta c_x(x^*;\gamma) - 1 - t + \beta c_x(x^*;\gamma) - \beta c_x(x^*;\gamma) - c_x(x^*;\gamma\,) + c_x(x^*;\gamma\,) = 0$$

$$v_x(x^*;\rho) - 1 - t - \beta c_x(x^*;\gamma) - c_x(x^*;\gamma\,) + c_x(x^*;\gamma\,) = 0$$

$$v_x(x^*;\rho) - c_x(x^*;\gamma\,) - 1 - t - \beta c_x(x^*;\gamma) + c_x(x^*;\gamma\,) = 0$$

$$v_x(x^*;\rho) - c_x(x^*;\gamma\,) - 1 - t + (1-\beta)c_x(x^*;\gamma) = 0$$

$$\therefore -t + (1-\beta)c_x(x^{**};\gamma) = 0$$

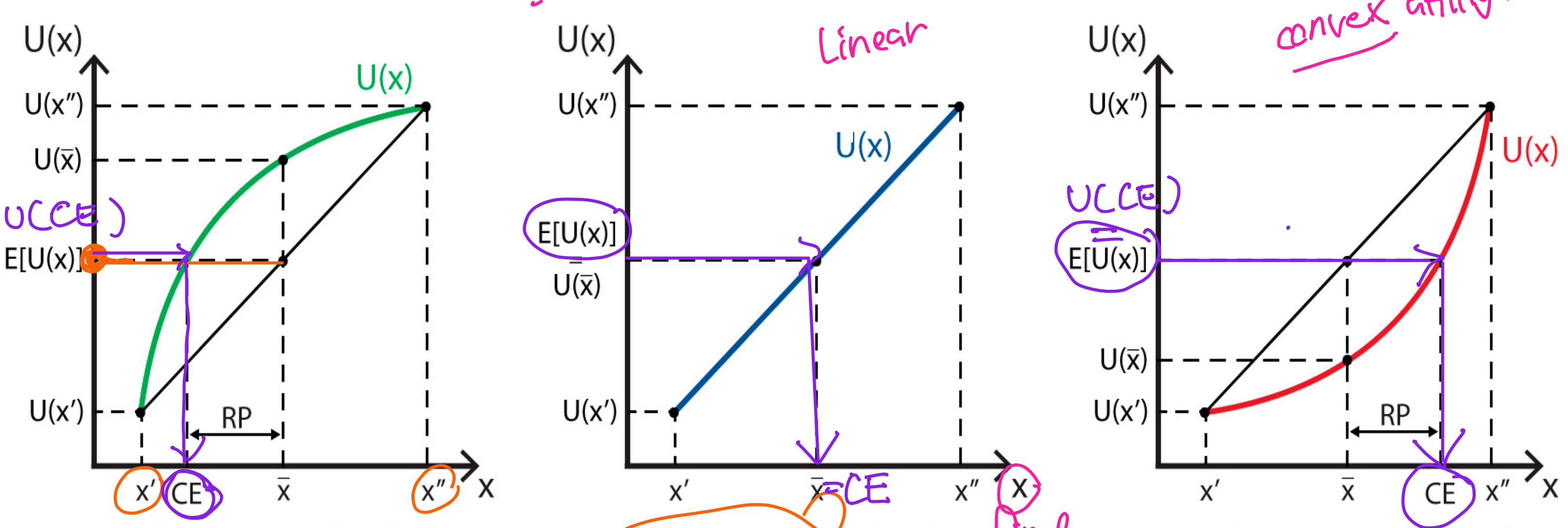
$$\therefore t^{**} = (1-\beta)c_x(x^{**};\gamma)$$

Ideal behavior x^{**} satisfies $v_x(x^{**};\rho) - c_x(x^{**};\gamma) - 1 = 0$

Recap:Risk aversion&Certainty equivalent

$$U(CE) = E[U(x)]$$

The **certainty equivalent** is an amount of money that provides equal utility to the random payoff of the gamble. The certainty equivalent is labeled *CE* in the figure. Note that *CE* is less than the expected outcome, if the person is risk averse. This is because risk-averse individuals prefer the expected outcome to the risky outcome.



Risk averse individual	Risk neutral individual	Risk loving individual
$E[U(x)] < U(\bar{x})$	$E[U(x)] = U(\bar{x})$	$E[U(x)] > U(\bar{x})$
$CE < \bar{x}$	$CE = \bar{x}$	$CE > \bar{x}$

Source: 1, 2

Recap

Suppose you have initial wealth w,
Consider lotteries $X = (x, p; 0, 1-p)$; i.e. $X = (x, p)$
How will you evaluate lottery X ?

Expected Value Theory

EV of the gamble

$EV =$ p (W + x) + 1-p (W + 0) $= \underbrace{W}_{\text{final wealth}} + \overbrace{px}^{\text{EV of the gamble}}$

EV after taking into account the gamble

Expected Utility Theory

$EU =$ p U(W + x) + (1-p) U(W)

Prospect theory

normally is 0 from v(0) = 0

$V(X, p) =$ π(p)v(x) + π(1-p)v(0) $v(.)$ at the reference point is usually normalized to 0.

π(·)

Recap

Suppose you have initial wealth w,
Consider two lotteries $X = (x, p; 0, 1 - p)$ and $Y = (y, q; 0, 1 - q)$
You will prefer lottery X over lottery Y IF….

Expected Value Theory

$$p(w + x) > q(w + y)$$

Expected Utility Theory

$$pU(w + x) + (1 - p)U(w) > qU(w + y) + (1 - q)U(w)$$

Prospect theory

$$\boxed{\pi(p)v(x) > \pi(q)v(y)}$$

Prospect Theory: Editing Stage

organize & reformulate the problem

more complicated / unsimplified gambles

What is going on in editing stage? Taking an objective prospect $(\hat{x}_1, \hat{p}_1; \ldots; \hat{x}_m, \hat{p}_m)$ and transforming it into an object for evaluation $(x_1, p_1; \ldots; x_n, p_n)$ by:

- Coding: code outcomes as gains & losses relative to reference point.
- Combination: e.g., (100, .5; 100, .5) replaced with (100, 1)
- Segregation: e.g., (100, .5; 200, .5) replaced with (100, 1) plus (100, 0.5 ; 0, 0.5)
- Cancellation: discard shared components.
- Simplification: rounding of probabilities. 9.999% to win \$100 → 10% to win \$100
- Eliminating dominated alternatives. ⇒ eliminating inferior choice

Prospect theory: Evaluation stage

A prospect can be written as $(x, p; y, q)$ with $p + q \leq 1$.

Note: $p + q < 1$ implies prospect yields 0 with probability $1 - p - q$.

A person evaluates a prospect $(x, p; y, q)$ according to the functional

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(y)$$

Note: Nowadays, it might be best to take Prospect theory as a theory for simple prospects with at most two non-zero outcomes.

Prospect Theory: Value Function

Three key features of the value function $v(.)$: gain / loss

- The carriers of value are changes in wealth ~~($v(.) = 0$)~~. $v(0) = 0$
- Diminishing sensitivity to the magnitude of changes:

$v(x)$ is concave in the gain domain, $v(x)$ is convex in the loss domain

$v''(x) < 0$ for $x > 0$, $v''(x) > 0$ for $x < 0$

- Loss aversion: losses loom larger than gains.

$\frac{\Delta v}{\Delta x}$

$$\frac{v'(x) \text{ when } x < 0}{v'(x) \text{ when } x > 0} > 1$$

if $0 < y < x$

$$(y, 0.5; -y, 0.5) \succ (x, 0.5; -x, 0.5)$$

ex $y = 1{,}000$ ฿ , $x = 2{,}000$ ฿

Observation A
confirm this with the part

$\frac{\Delta v}{\Delta x}$

$\frac{\text{marginal pain from loss}}{\text{marginal pleasure from gain of equal size}} > 1$

linear value f

Prospect Theory: Value Function

Two common functional forms for the value function:

Tversky & Kahneman (1992) → diminishing sensitivity & loss aversion

$$v(x) = \begin{cases} x^{\alpha} & if\ x > 0 \\ -\lambda(-x)^{\beta} & if\ x < 0 \end{cases}$$

,where $\alpha, \beta \in (0,1]$ and $\lambda \geq 1$

Two-part linear assume no diminishing sensitivity, have loss aversion

$$v(x) = \begin{cases} x & \text{if } x \geq 0 \\ \lambda x & \text{if } x < 0 \end{cases} \qquad \lambda > 1$$

λ is the coefficient of loss aversion

Rabin's calibration (Rabin, Econometrica 2000)

Small stake risk aversion

People tend to dislike risky prospects even when they involve an expected gain.

E.g. A 50-50 gamble of losing \$100 vs. gaining \$105.

(\$105, 50% ; −\$100, 50%)

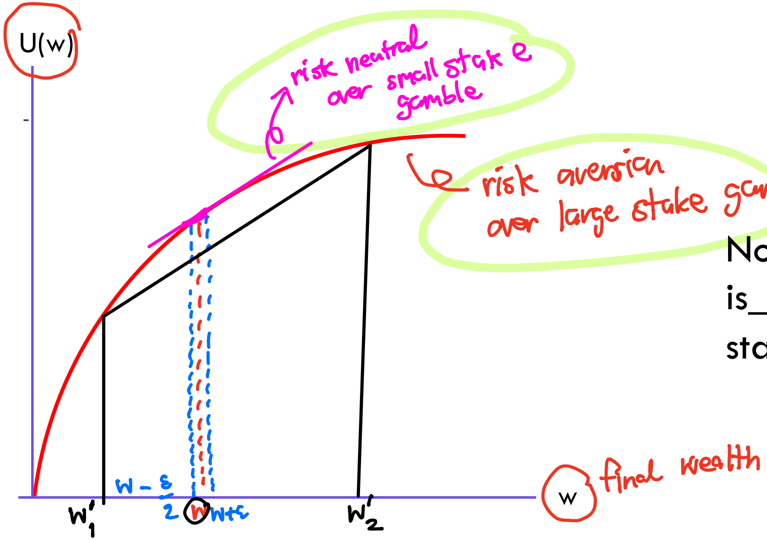
Standard explanation:

EU theory with a concave utility function.

Rabin's Point:

This explanation doesn't work, because according to EU theory, anything but virtual risk neutrality over modest stakes implies manifestly unrealistic risk aversion over large stakes.

Rabin's calibration(Rabin, Econometrica 2000)



Normal concave utility function is risk-neutral for very small stake gambles.

We can use 'loss aversion' to explain risk averse behavior in small stake gamble.

In mixed gambles $\mathrm{L} = (x, p; y, q), x > 0, y < 0$, where both a gain and a loss are possible, loss aversion causes extremely risk-averse choices. in small stake gambles

small in magnitude

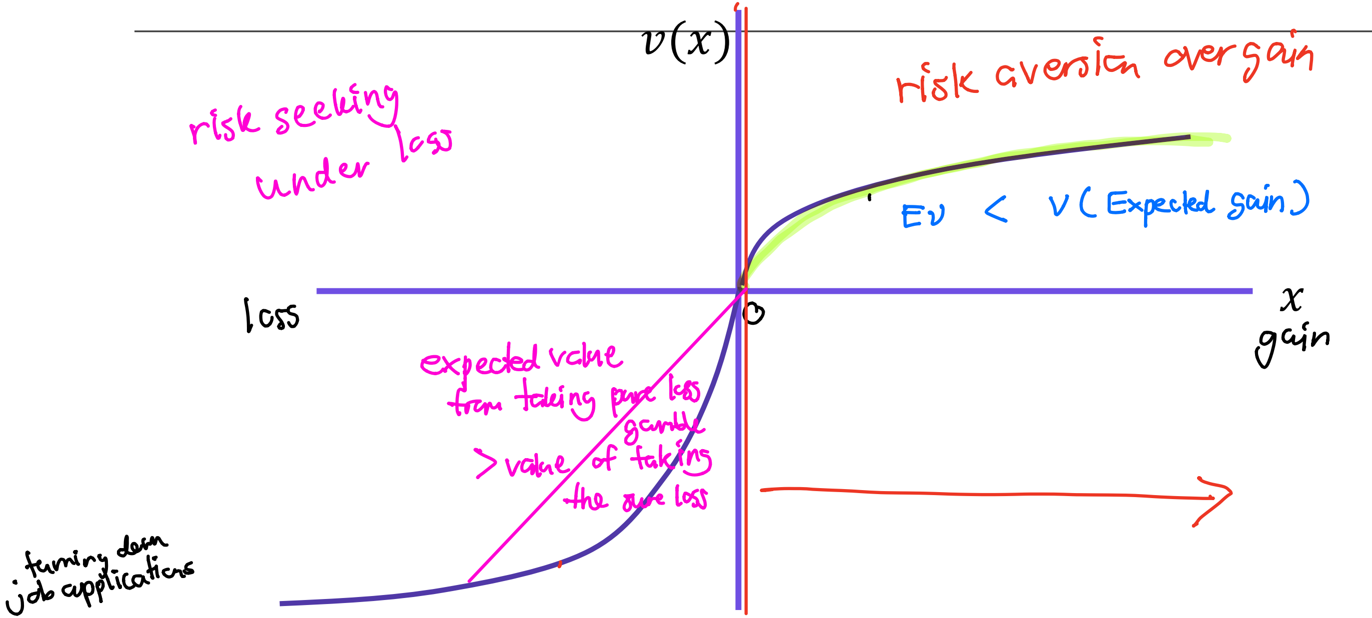
Risk aversion over gain & Risk seeking over loss

In a situation where a __________is compared to a gamble with (highly) probable larger loss, diminishing sensitivity cause risk_______.

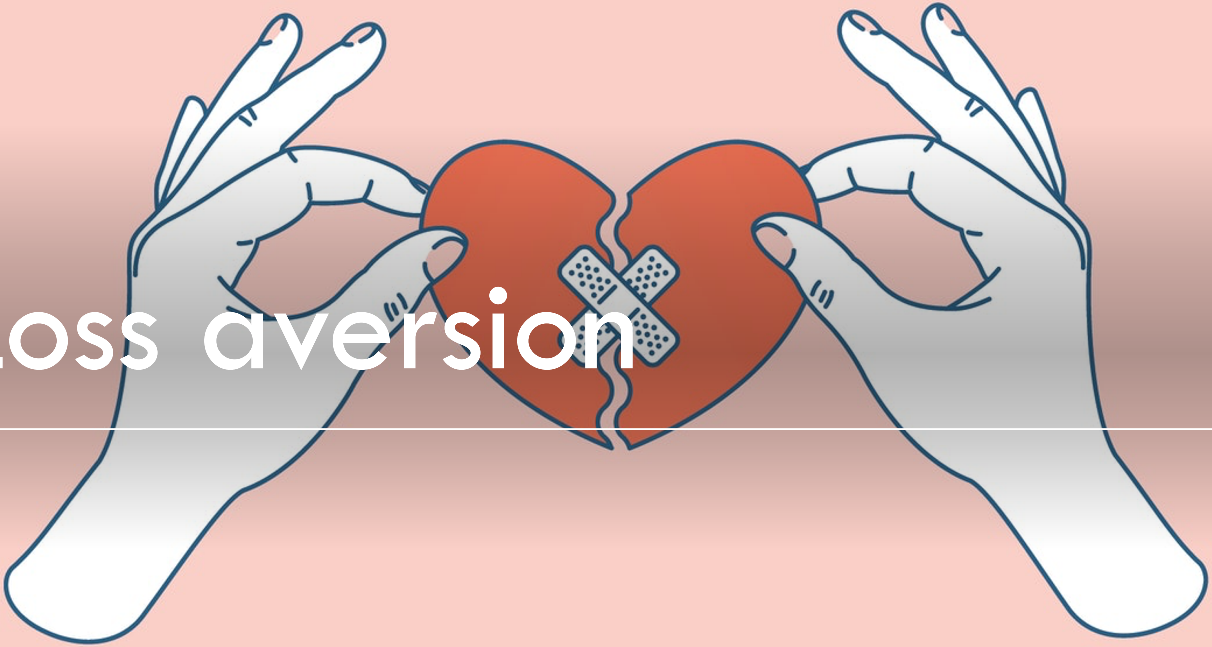
In a situation where a _________is compared to a gamble with (highly) probable larger gain, diminishing sensitivity cause risk________.

At the same level of wealth, an individual can both be risk averse and risk seeking.

Value function



Loss aversion



Prospect Theory: Value Function Measuring Loss Aversion

How loss averse is the average person?

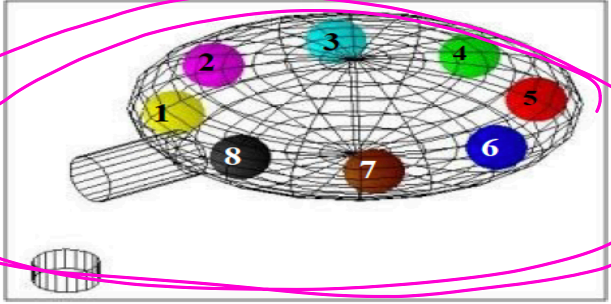
Would you accept the following bet? A coin toss where:

If tails, lose $100, heads win $150?

If tails, lose $100, heads win $160?

If tails, lose $100, heads win $170? ...

asked until a person accept the coin toss



Win € 20 if one of the following balls is extracted:

1 **2** **3** **4**

If one of the following balls is extracted, then:

5 **6** **7** **8**

Choose the gamble

reject the gamble.

Lose € 20	O	O	€ 0 for sure	row 1
Lose € 19	O	O	€ 0 for sure	row 2
Lose € 18	O	O	€ 0 for sure	row 3
Lose € 17	O	O	€ 0 for sure	
Lose € 16	O	O	€ 0 for sure	
Lose € 15	O	O	€ 0 for sure	
Lose € 14	O	O	€ 0 for sure	
Lose € 13	O	O	€ 0 for sure	
Lose € 12	O	O	€ 0 for sure	
Lose € 11	O	O	€ 0 for sure	
Lose € 10	O	O	€ 0 for sure	

Prospect Theory: Value Function Measuring Loss Aversion

λ

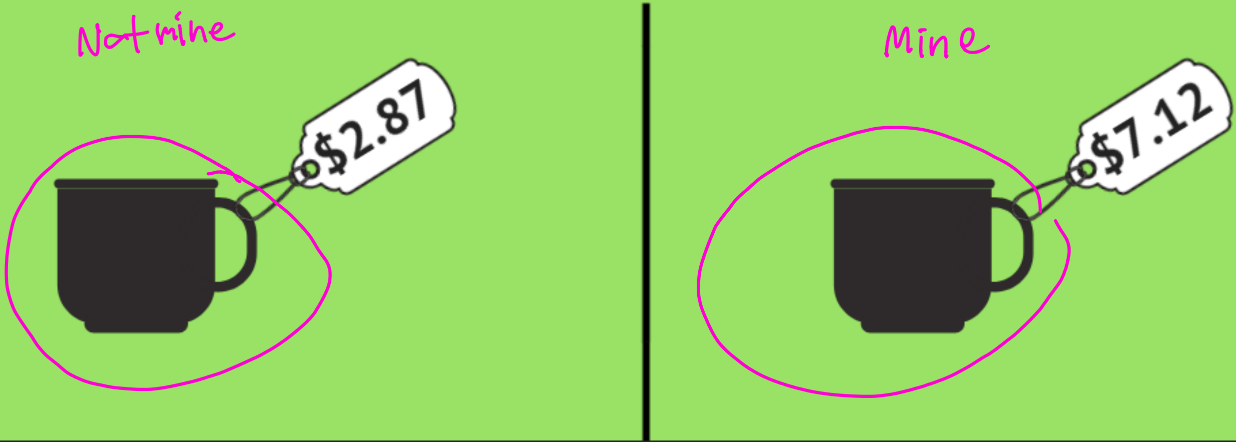
The coefficient of loss aversion is the ratio of the amount

you'd need to win (to accept the bet) and the amount you could potentially lose

For coin toss above, typical response is to be indifferent at gain = $200

Loss aversion coefficient or ratio is generally found in range 1.5 to 2.5.

The Endowment Effect



Around the same time Kahneman and Tversky were working on their model of Prospect Theory, economist Richard Thaler was noticing (and writing about) some of his own "anomalies".

RICHARD THALER

Willingness to Pay(WTP) vs. Willingness to Accept(WTA)

- Standard economic theory says that an individual has a single value (i.e. some price p) that she associates with any good

➢ If she is given the opportunity to buy the good at (or below) that price, she will pay it. The price is her Willingness to pay (WTP)

➢ Similarly, if someone offers to buy the good from her at (or above) that price, she will accept it. The price is her Willingness to accept (WTA)

➢ Standard economic theory says that WTP = WTA

Willingness to Pay(WTP) vs. Willingness to Accept(WTA)

In reality, we often observe behavior that suggests otherwise…

Example 1: Thaler's professor was a wine collector who frequently shopped at wine auctions

- He never paid more than $35 for a bottle ("willingness to pay")
- …but he never sold them for less than $100 ("willingness to accept")

Willingness to Pay(WTP) vs. Willingness to Accept(WTA)

Example 2: You buy a ticket to see Chelsea at Stamford Bridge, and you pay the face value ($300) for your ticket

- You would have actually paid up to $500 (WTP)
- ...but you wouldn't sell the ticket online for less than $1000 (WTA)

The Endowment Effect

Why is there the gap between willingness to pay and willingness to accept?

Thaler (1980) attributes gap to what he coined **the endowment effect.**

" People tend to value an object more highly when they own it than when they do not. "

The Endowment Effect

- Originally modeled as an extra utility gain from having/owning an object (rather than getting/receiving an object) – individuals value an object more when they own it
- Related to loss aversion – the disutility from giving up an item is larger than the utility from receiving that same item

Endowment Effect: Mug Experiment

Kahneman, Knetsch, and Thaler (1990) test these ideas in their famous "mug experiments" at Cornell University.

Typical Experiment: Class of 50 - 75 economics students.

After students sit down, half of them are given a coffee mug.

Endowment Effect: Mug Experiment
Design

	
Potential "Sellers"	Potential Buyers
How much willing to accept for the mug?	How much willing to pay for the mug?
Each seller were given a coffee mug.	Each buyer were shown a coffee mug.

If there is a WTP > WTA in the group, they trade.

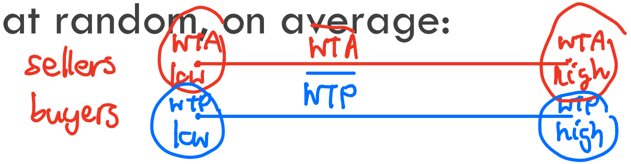
Endowment Effect: Mug Experiment

Prediction

What does standard economic theory predict? How many will trade?

When markets clear, mugs will be owned by those who value them the most

Since mugs were assigned at random, on average:



- $\overline{WTP} = \overline{WTA}$
- If roughly half of "mug lovers" was given mugs, and half wasn't, then about half of the mugs should trade.

Endowment Effect: Mug Experiment

Result

- Cornell bookstore price is $6.00 for the mug

- Students might value the mug more or less than this price, but standard theory says that, on average, we should find WTP = WTA

- Results:

Buyers: (median) WTP = $2.25

Sellers: (median) WTA = $5.75

This result is even for goods they just received (and didn't necessarily want).

This result was claimed as an endowment effect.

Endowment Effect: Mug Experiment

Result

- Add a "Chooser" group: elicit the amount of money at which they would be indifferent between receiving a mug and receiving the money

if price $5

Sellers (were given mugs and have a choice to sell a mug) :

+$5

Sellers' choice: (mug , $0) vs. (no mug , money from selling)

Buyers(were not given mugs and have a choice to buy a mug):

−$5

Buyers' choice (mug , − money paid for mug) vs. (no mug , $0)

Choosers(have a choice to choose between a mug or money):

Choosers' choice: (mug , $0) vs. (no mug , $5)

- But sellers and choosers have the EXACT same choice

Endowment Effect: Mug Experiment Design

Elicit people's reservation values (or reservation prices):

A buyer's WTP is the maximum amount she is willing to pay to obtain the object.

A seller's WTA is the minimum amount she is willing to accept to part with the object.

If there is a WTP > WTA in the group, they trade.

Sellers [Buyers]	I will sell [I will buy]	I will keep mug [I will not buy]
If the price is \$0.00:		
If the price is \$0.50:		
…		
If the price is \$9.00:		
If the price is \$9.50:		

Choosers: **indicate for each price whether they want the mug or the money.**

Endowment Effect: Mug Experiment

Result

Results:

WTA to sell the mug

Sellers: $7.12

WTP to buy the mug

Buyers: $2.87

Choosers: $3.12 I value the mug at $3.12

Coefficient of Loss Aversion

Measurement

- Earlier, we measured the coefficient of loss aversion from answers to whether you would accept a bet with a 50% chance to win X, and a 50% chance to lose Y, where:

Coefficient of Loss Aversion = $\frac{X}{Y}$ ≈ 2–3 for the smallest X given a fixed Y, OR for the largest Y given a fixed X

value of mug × value of loss

- We can also measure the coefficient of loss aversion using the endowment effect experiment:

Coefficient of Loss Aversion = $\frac{\text{WTA}}{\text{WTP}}$

WTA ≈ value of mug × value of loss

WTP ≈ value of mug

Using data from the experiments above =5.75/2.25 = 2.6