Practice problem set 5 EE320 Semester 1/2017

Chapter 7: Multivariate calculus: *basis derivative and applications*

Question 1: Determine Z_x, *and* Z_y

a. $Z = (x^2y + 2)^2$
b. $Z = x^{1/2} y^{1/3}$
c. $Z = x\,y - ln(xy)$

Question 2:

Consider a function $Z = f(x, y) = x^3 + 3x^2y + 6xy^2 - y^3$

a. Evaluate the gradient matrix when x = 0 and y = 2
b. Derive the Hessian matrix.
c. Evaluate the value of the Hessian matrix where x = 2 and y =3

Question 3:

a. Given that $Z = \frac{x^3 - y^3}{x^2y^2}$, show that $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y} = -Z$

b. Given that $Z = \frac{x - y}{x + y}$, find $x\frac{\partial Z}{\partial x} + y\frac{\partial Z}{\partial y}$ =?

c. If z = $2x^2y + 3xy + y^2$ where $x = r^2 + 2rs$ and $y = 2r - 4s$, then by means of the chain rule, (c.1a) find ∂z/∂s and ∂z/∂r ; (c.2) evaluate when r = 1 and s = 0

d. For $2x^2 + 3y^2 + 2z^2 = 16$, evaluate ∂z/∂y when x = 1, y = 2, z = -1.

e. Given that $ln\,(x + y + z) + xyz = ze^{x+y+z}$, evaluate ∂z/∂x when x = 0, y = 1, z = 0.

Question 4

Suppose that the demand of a product is given by,

$$Q_x = 100 - 4P_x - \frac{50}{\sqrt{P_y}} + 0.5I^2 .$$

where P_x is the price of good X, P_y is the price of good Y, and I is the level of income.

Consider the following problems

a. What is the relationship between good x and good y? Are they substitute product/complementary product? Show your results using the partial derivative

b. Is the product X considered an inferior product?

c. What is the level of quantity demanded if $P_x = 10,\ P_y = 25\ \ and\ I = 10$?

d. Calculate the own-price elasticity of demand, and evaluate the value when $P_x = 10,\ P_y = 25\ \ and\ I = 10$.

e. Calculate the cross-price elasticity of demand when $P_x = 10,\ P_y = 25\ \ and\ I = 10$.

f. Calculate income elasticity of demand when $P_x = 10,\ P_y = 25\ \ and\ I = 10$. Is the product a necessary or luxurious product?

Question 5: Consider a simple market model where

Demand: $q = -p + \sqrt{I}$

Supply: $q = 3p - w^2$

where q is quantity of output, p is price, I is the level of income, and w is the price of factor input

a) Derive the solution of all the endogenous variables.

b) Use the partial derivative to show that, under the equilibrium, an increase in W causes a decrease in quantity, but an increase in price.

c) Predict the impact of an increase in income on equilibrium quantity and price.

d) Suppose initial wage and income are \$2 and \$4 respectively. Find the approximate changes in equilibrium price and equilibrium quantity if income and wage both equally increase by \$1.

Question 6: Find the total differentials for the following functions:

a. $z = 4x^3 - 13xy - 6y^5$

b. $z = (2x^2 - y)(3x - 4y^3)$

c. $z = 8x^{\frac{1}{2}}y^{\frac{1}{4}}w^{\frac{1}{4}}$

Question 7: Suppose that Mr. A's utility depends on two commodities: x_1 and x_2. His utility function is given by

$$U = 25x_1 + 27x_2 - 3x_1^2 - 7x_1x_2 - 4x_2^2$$

(a) Determine the marginal utility function of each commodity.

(b) Find the marginal rate of substitution (MRS) between the two commodities when $x_1 = 2$ and $x_2 = 1$.

Question 8: Given the equation for the production isoquant

$$18[0.2K^{-0.4} + 0.8L^{0.4}]^{-2.5} = 1936$$

Use the implicit function rule to find the marginal rate of technical substitution (MRTS) of L for K.

Question 9: The demand for money, *M*, in the United States for the period 1929-1952 has been estimated as

$$M = 0.14Y + 76.03(r-2)^{-0.84} \quad , \qquad (r > 2)$$

where *Y* is the annual national income, and *r* is the interest rate measured in percent per year. Find $\frac{\partial M}{\partial Y}$ and $\frac{\partial M}{\partial r}$ and discuss their signs.

Question 10:

Suppose that a firm produces $Q = f(L) = L^{1/2}$ units of commodity using L units of labor. Assume that $f'(L) > 0$ and $f''(L) < 0$, so f is strictly increasing and strictly concave. If the firm gets P baht per unit produced and pays w baht for a unit of labor, write down the profit function, and find the first-order condition for profit maximization at $L^* > 0$. Show that

Question 11:

Let the demand for a commodity be

$$Q_d = D(P, Y_0, t_0), \qquad \frac{\partial D}{\partial P} < 0;\ \frac{\partial D}{\partial Y_0} > 0;\ \frac{\partial D}{\partial t_0} < 0$$

$$Q_s = S(P, T_0), \qquad \frac{\partial S}{\partial P} > 0;\ \frac{\partial S}{\partial T_0} < 0$$

where P is the price, Y_0 is the consumer's income, t_0 is the taste for the commodity and T_0 is the tax on the commodity. All derivatives are continuous. Use implicit function rule to find $\frac{\partial P^*}{\partial t_0}$, $\frac{\partial P^*}{\partial T_0}$, $\frac{\partial Q^*}{\partial Y_0}$, and $\frac{\partial Q^*}{\partial T_0}$. Discuss their economic implications.

Question 12:

A joint-cost function is defined implicitly by the equation

$$c + \sqrt{c} = 12 + q_A\sqrt{9 + q^2_B}$$

where c is denoted as the total cost for producing q_A and q_B units of product A and B, respectively.

a) If $q_A = 6$ and $q_B = 4$, find the corresponding value of c.
b) Determine the marginal costs with respect to q_A and q_B, when $q_A = 6$ and $q_B = 4$.

Question 13:

Suppose a production function is given by $P = \frac{kl}{k+l}$

a. Determine the marginal productivity functions.
b. Show that when $k = l$, the marginal productivities are equal.

Question 14:

Suppose the demand equations for related products A and B are

$$q_A = e^{-(p_A+p_B)} \quad \text{and} \quad q_B = \frac{16}{p_A^2\, p_B^2}$$

where q_A and q_B are the number of units of A and B demanded when the unit prices (in thousands of dollars) are p_A and p_B , respectively.

a. Classify A and B as competitive, complementary, or neither.
b. Calculate elasticity of demand for good A and good B, respectively. (Get me all whatsoever you can do for the elasticities.)
c. If the unit prices of A and B are $1000 and $2000, respectively, estimate the change in the demand for A when the price of B is decreased by $20 and the price of A is held constant.

Question 15:

Suppose the cost c of producing q_A units of product A and q_B units of product B is given by $c = (3q_A^2 + q_B^3 + 4)^{1/3}$. And the coupled demand functions for the products are given by

$$q_A = 10 - p_A + p_B^2 \qquad \text{and} \qquad q_B = 20 + p_A - 11p_B$$

Use a chain rule to evaluate $\frac{\partial c}{\partial p_A}$ and $\frac{\partial c}{\partial p_B}$ when $p_A = 25$ and $p_B = 4$.
