

More Past Exams

2. Use **Cramer's rule** to solve the following system of linear equations.

(2 marks)

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & 4 \\ 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

3. a) Let  $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ , find determinant of  $\mathbf{A}$ , adjoint of  $\mathbf{A}$  and inverse of  $\mathbf{A}$ .

b) Let  $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix}$ ,  $(\mathbf{BC}^T - \mathbf{I})^{-1} = \mathbf{A}^T$ , find the matrix  $\mathbf{C}$ .

(3 marks)

3. a) **Use cofactor expansion together with row operation** to determine

determinant of  $\underline{\mathbf{A}}$  where  $\underline{\mathbf{A}} = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$  (leave your answer in term of  $a, b$  and  $c$ ).

b) If  $a = -2$ ,  $b = 3$  and  $c = 1$ , determine  $\det(\mathbf{A})$ ,  $\det(2\mathbf{A})$ ,  $\det(-\mathbf{A})$ ,  $\det(\mathbf{A}^{-1})$  and  $\det(\mathbf{A}^2)$

c) If  $\det(\mathbf{A}) = 0$  and  $c = 1$ , give all possible values for  $a$  and  $b$ ?

(6 marks)

5. Suppose  $\underline{\mathbf{A}}$  is a 4 by 3 matrix in which  $\underline{\mathbf{A}} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 4 \end{bmatrix}$ ,  $\underline{\mathbf{A}} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \end{bmatrix}$  and

$\underline{\mathbf{A}} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ . Determine the matrix  $\underline{\mathbf{A}}$ . (7 marks)

6. Given  $\underline{\mathbf{A}} = \begin{bmatrix} 1 & 0 & -2 & 2 \\ 4 & -2 & 1 & 8 \\ 1 & 2 & -10 & 2 \\ -1 & 0 & 2 & -1 \end{bmatrix}$ ;  $\underline{\mathbf{B}} = \begin{bmatrix} -11 & 2 & 2 & -2 \\ -41/2 & l & 9/2 & 0 \\ k & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$ ;  $\underline{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$

- a) What are values of  $k$  and  $l$  if  $\underline{\mathbf{B}}$  is an inverse of  $\underline{\mathbf{A}}$ ?  
 b) Determine  $\det \underline{\mathbf{A}}$  and  $\det \underline{\mathbf{C}}$ ?  
 c) If  $\det (\underline{\mathbf{A}} \underline{\mathbf{B}} \underline{\mathbf{D}}^3 \underline{\mathbf{C}}^{-1} \underline{\mathbf{B}}^T) = 36$ , find  $\det \underline{\mathbf{D}}$ .

d) Solve for  $\underline{\mathbf{x}}$  if  $(\underline{\mathbf{A}}^T \underline{\mathbf{B}}^T \underline{\mathbf{C}}^{-1})^T \underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ .

(11 marks)

4. Consider the system of equations

$$-3x + 4y = 8$$

$$6x + ty = s$$

where  $t$  and  $s$  are real numbers

- (a) Write the matrix equation for this system.  
 (b) Find values of  $s$  and  $t$  so that the system has exactly one solution.  
 (c) Find values for  $s$  and  $t$  so that the system has no solutions.

(d) Find values for  $s$  and  $t$  so that the system has infinitely many solutions. For this case (case d)), solve the system of equations above. Also write down the solution **in vector form**.  
 (9 marks)

7. If you know that  $\det \mathbf{A} = 6$ , what is the determinant of  $\mathbf{B}$ ?

$$\det \mathbf{A} = \begin{vmatrix} \text{row 1} \\ \text{row 2} \\ \text{row 3} \end{vmatrix} = 6 \quad \det \mathbf{B} = \begin{vmatrix} \text{row 3} + \text{row 2} + \text{row 1} \\ \text{row 2} + \text{row 1} \\ \text{row 1} \end{vmatrix} = ?$$

(7 marks)