# EE312 Macroeconomics, 2/2013 (Sec. 046402)
# Chapter 4. Solow Growth Model

**Practice Questions**

1. In solow growth model, explain the effect of a decrease in population growth rate on all endogenous variables. Explain and comment..

- Define model settings

| | | | |
|---|---|---|---|
| $Y$ | = | ........................ | production function |
| $S$ | = | ........................ | saving is a constant fraction of Y |
| $C$ | = | ......................... | |
| $K'$ | = | ......................... | capital stock next period is equal to capital after depreciation plus investment |

- Equilibrium conditions

$$S = \text{..............................................}$$

$$K' = \text{..............................................}$$

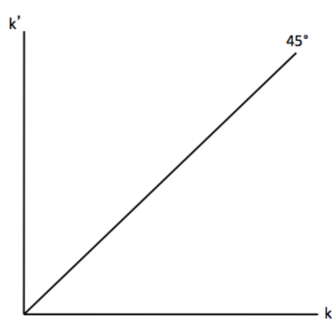
$$\frac{K'}{N} = \text{..............................................}$$

$$k'(1+n) = \text{..............................................}$$

$$k' = \qquad\qquad\qquad\qquad (*)$$

.

- Steady state




- From equation (*), we plot $k'$ against $k$
- To the left of k*, k'> k so that k is .................
- To the right of k*, k'< k so that k is ................
- At E, k = k'= k* so that k* is steady. k*; steady-state capital per worker.

$$k^* = \text{....................................................} \quad (**)$$

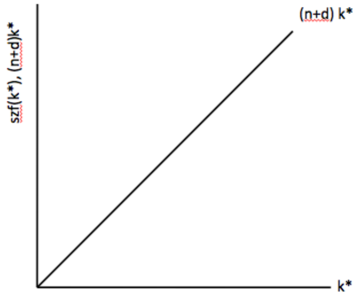
$$(n+d)k* = \text{....................................................} \quad (***)$$

steady state.................... = steady state .......................

At steady state, $k^*$ is constant. Since, $y^* = f(k^*)$, $y^*$ is constant. All aggregate variables ($Y = .....y^*, C = (1-s)....., K = ......k*, I = s.....$) are growing at ...... (population growth rate).

Suppose population growth rate is initially at $n_1$. Initially, All aggregate variables ($Y = .....y^*, C = (1-s)....., K = ......k*, I = s.....$) are growing at ...... (population growth rate).

- Analyze the effect of a decrease in population growth rate on all endogenous variables




- From (***), plot steady state investment and steady state saving against steady state capital per worker.
- As population growth rate decreases from $n_1$ to $n_2$, .............. rotates ..............
- **steady-state capital per workers,** $k^*$..................

**Since** $k^*$..........., $y^*$............ **and** $c*$................ (**per worker variables** ............ )

At steady state, $k^*$ is constant. Since, $y^* = f(k^*)$, $y^*$ is constant. Now, all aggregate variables ($Y = .....y^*, C = (1-s)....., K = ......k*, I = s.....$) are growing at ...... (new population growth rate).

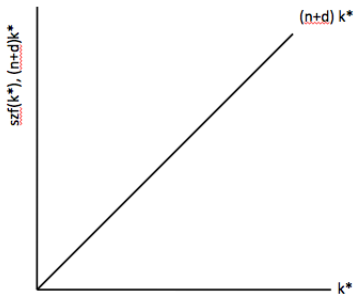
**Growth rates of all aggregate variables** ...................... from $n_1$ to $n_2$.

- **Comment :**
  - Intuition : As $n$ decreases, $k^*$ increases. As capital per worker rises, output per worker rises. The reason is that when labor force grows at a lower rate, the current labor force faces an easier task in bulding capital for the next period's consumers, who are a proportionately a smaller group. Thus, output per worker and capital per worker increases at steady state.
  - This shows that the lower growth in aggregate income neednot be associated, in the long run, with lower income per worker.
  - consistent with the empirical fact: the higher population growth, the lower output per worker. High population growth corresponds with low living standards.

**Practice Questions (cont.)**

2. In Solow growth model, suppose that total factor productivity decreases. Analyze the effects on the economy especially consumption, capital and output per worker at steady state. Explain and comment.

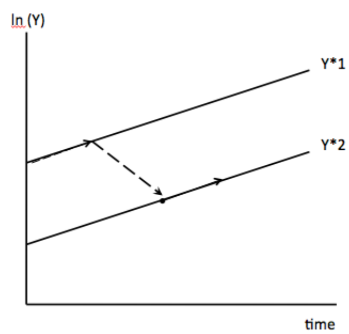
- Analyze the effect of a decrease in total factor productivity on all endogenous variables




- From (***), plot steady state investment and steady state saving against steady state capital per worker.
- As total factor productivity decreases from $z_1$ to $z_2$, ............. rotates .............
- **steady-state capital per workers,** $k^*$.................

**Since** $k^*$..........**,** $y^*$........... **and** $c*$............... (**per worker variables** ........... )

At steady state, $k^*$ is constant. The new steady state capital per worker is equal to $k_2^*$ which is ..... $k_1^*$.




- [note: we plot natural log y against time. The slope of $\ell n Y$ is equal to $\dfrac{d\ell nY}{dt} = \dfrac{1}{Y}\dfrac{dY}{dt} = \dfrac{1}{Y}\dfrac{\Delta Y}{\Delta t}$ = growth rate of Y]
- Y*1 indicates the original growth path.
- Y*2 indicates the new growth path
- Note that the two growth path have the same slope. Growth rate of aggregate output is equal to population growth rate at steady state.

- Since $k^*$ decreases from $k_1^*$ to $k_2^*$, $f(k^*) = y^*$ decreases and $Y^*$ decreases as a consequence.
- **In transition to the new growth path,** the growth rate of $Y$ (and all aggregate variables) decreases temporary. $k^*$ reduces temporary.
  $k$ ....... temporary, $y$ ............ and $c$ ............ [ $y = f(k)$, $c = (1-s)y$ ] temporary
  growth rate of $Y$ ......... temporary (the growth rate of aggregate variables is temporary .......... than $n$ )
- **After the economy have arrived the new growth path,** $k$ **remains constant at** $k_2^*$ **and aggregate output continue to grow at rate** ......
  All aggregate variables ($Y = .....y^*, C = (1-s)....., K = ......k*, I = s.....$) grows at rate ...........

3. In Solow growth model, suppose that the MPK increases for each quantity of the capital input.
   (a) Show the effects of this on the aggregate production function
   (b) Using a diagram, determine the effect on the quantity of capital per worker
   (c) Explain your results

[For the marginal product of capital to increase at every level of capital, the shift in the production function is equivalent to an increase in total factor productivity.]

Analysis of steady state :

- ................. rotates .................. , $k^*$ ........ from $k_1^*$ to $k_2^*$ .
- $k$ ....... temporary, $y$ ............ and $c$ ............ [ $y = f(k)$, $c = (1-s)y$ ] temporary
- growth rate of $Y$ ......... temporary (the growth rate of aggregate variables is temporary higher than $n$ )

- For a given savings rate, more effective capital implies more savings, and in the steady state there is more capital and more output.
- After the economy have arrived the new growth path, $k$ remains constant at $k_2^*$ and aggregate output continue to grow at rate ......
- [Note that if the economy can increase effective capital (or increase total factor productivity) continuously, Y will grows at growth rate $> n$. $k^*$ and $y^*$ keep growing. Sustained increases in z cause sustained improvements in y*. ]
- "An increase in a country's propensity to save or a decrease in the labor force growth rate imply one time increase in a country's standard of living, but **there can be unbounded growth in the standard of living if and only if total factor productivity increases.**" PP. 269 Williamson

4. In Solow growth model, suppose that saving rate decreases. Analyze the effects on the economy especially consumption, capital and output per worker at steady state. Explain and comment.

    - .................. rotates ................... , $k^*$ ........ from $k_1^*$ to $k_2^*$ .
    - $k^*$ ....... temporary
    - growth rate of $Y$ .......... temporary
    - For a given savings rate, more effective capital implies more savings, and in the steady state there is more capital and more output.
    - After the economy have arrived the new growth path, $k$ remains constant at $k_2^*$ and aggregate output continue to grow at rate ......
    - Changes in savings rate causes "level effect", but no long run growth effect.
    - A decrease in a country's propensity to save implies one time decrease in a country's standard of living ($y^*$).

5. In Solow growth model, suppose that depreciation rate increases. Analyze the effects on the economy especially consumption, capital and output per worker at steady state. Explain and comment.

    - An increase in the depreciation rate acts in much the same way as an increase in ..................................................
    - .................... rotates .......................... , $k^*$ ........ from $k_1^*$ to $k_2^*$ .
    - $k^*$ ....... temporary
    - More of current savings is required just to keep the amount of capital per capita constant. In equilibrium output per capita and capital per capital decrease.
    - growth rate of $Y$ .......... temporary
    - After the economy arrives the new growth path, $k$ remains constant at $k_2^*$ and aggregate output continue to grow at rate ......

6. Suppose that the economy is initially in a steady state and that some of the nation's capital stock is destroyed because of a natural disaster or a war.

   (a) determine the long-run effects of this on the quantity of capital per worker and on output per worker.

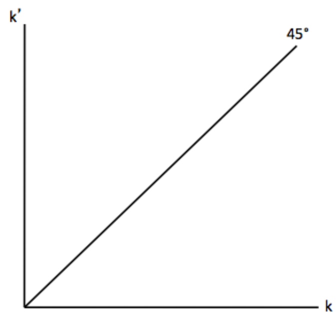
   (b) In the short-run does aggregate capital stock grow at a rate higher or lower than growth rate of the labor force

   (c) After World War II, growth in real GDP in Germany and Japan was very high. How do your results in parts (a) and (b) shed light on this historical experience.

      i. The long-run equilibrium is not changed by an alteration of the initial conditions. If the economy started in a steady state, the economy will return to the same steady state. If the economy were initially below the steady state, the approach to the steady state will be delayed by the loss of capital.

      ii. Initially, the growth rate of the capital stock will exceed the growth rate of the labor force. The faster growth rate in capital continues until the steady state is reached.

      iii. The rapid growth rates are consistent with the Solow model's predictions about the likely adjustment to a loss of capital.




7. Consider the effects of change in immigration law, which make it easier for immigrants to move to Canada (receiving work-permit and citizenship). More and more immigrants influx to the country from other troubled countries each year. Analyze the effects on the country's economy especially consumption, capital and output per worker at steady state. Explain and comment.

   The policy affects ...................................... in the long run.

   ..........................................................................................

8. The Solow growth model predicts that poor countries should grow faster than rich countries. Comment.

   ......................................................................................................................

   ......................................................................................................................

9. Consider a numerical example using the Solow growth model. Suppose that $F(K, N) = K^{0.5}N^{0.5}$, with $d = 0.1$, $s = 0.2$, $n = 0.01$, and $z = 1$ and take a period to be a year.

   (a) Determine capital per worker in the steady state.

   Steady state condition : $(n + d)k^* = szf(k^*)$

   Find $f(k)$. $f(k) = \frac{F(K,N)}{N} = \frac{K^{0.5}N^{0.5}}{N} =$ ...............

   $$\begin{aligned} (n+d)k^* &= szf(k^*) \\ (0.01+0.1)k^* &= 0.2 \times 1 \times \ldots\ldots. \\ \text{Then,} \quad 0.11k^* &= \\ \\ k^* &= 3.3058 \end{aligned}$$

   (b) Suppose that the economy is initially in the steady state that you calculate in part (a). s increases to 0.4. Determine capital per worker in the steady state. [$k^* = 13.32$]

**Note : Solow Growth Model**

- The law of motion for capital : $k^{'}=$ ....................................................
- Steady state condition :

  steady state investment $=$ steady state saving

  $=$

  .
- $Y*=y*N$ , $K^*=k^*N$ , $C^*=c^*N$ : $Y=C+S$ , $c=(1-s)Y$
- $z\uparrow$

  $\Rightarrow$ ....................... rotates .................

  $\Rightarrow$ $k^*$....... from $k_1^*$ to $k_2^*$, $y^*$ ............ , $c^*$...............

  $\Rightarrow$ In transition to the new growth path, $k^*$....... , $y^*$ ............ , $c^*$............... (jump). Hence, all aggregate variables $(Y=y^*N, K=k*N, C=c^*N, S=sY^*)$ grows at rate ...... $n$.

  $\Rightarrow$When the economy has arrived the new growth path $k^*$.............., therefore, $y^*, c^*$................... Hence, all aggregate variables $(Y=y^*N, K=k*N, C=c^*N, S=sY^*)$ grows at rate ...... $n$.
- $s\uparrow$

  $\Rightarrow$ ....................... rotates .....................

  $\Rightarrow$ $k^*$....... from $k_1^*$ to $k_2^*$, $y^*$ ............ [ $c^*$...............]

  $\Rightarrow$ In transition to the new growth path, $k^*$....... , $y^*$ ............ , (jump).[$c^*$................ ]. Hence, Y and K grows at rate ...... $n$.

  $\Rightarrow$When the economy has arrived the new growth path $k^*$.............., therefore, $y^*, c^*$................... . $c^*=s_{new}y^*$. Hence, all aggregate variables $(Y=y^*N, K=k*N, C=c^*N, S=sY^*)$ grows at rate ...... $n$.
- $n\uparrow$ from $n_1$ to $n_2$

  $\Rightarrow$ ....................... rotates .................

  $\Rightarrow$ $k^*$....... , $y^*$ ............ , $c^*$...............

  $\Rightarrow$ In transition to the new growth path, $k^*$....... from $k_1^*$ to $k_2^*$, $y^*$ ............ , $c^*$............... (jump). Hence, all aggregate variables $(Y=y^*N, K=k*N, C=c^*N, S=sY^*)$ grows at rate ...... $n_1$.

  $\Rightarrow$When the economy has arrived the new growth path $k^*$......................., therefore, $y^*, c^*$................... Hence, all aggregate variables $(Y=y^*N, K=k*N, C=c^*N, S=sY^*)$ grows at rate ...... which is ..... $n_1$.

  $\Rightarrow$Notice that per worker variables decrease but the growth rate of aggregate variable increases.

  - Intuition : As $n$ decreases, $k^*$ increases. As capital per worker falls, output per worker falls. The reason is that when labor force grows at a lower rate, the current labor force faces a harder task in bulding capital for the next period's consumers, who are a proportionately a largeer group. Thus, output per worker and capital per worker decreases at steady state.
  - This shows that the higher growth in aggregate income neednot be associated, in the long run, with higher income per worker.
  - consistent with the empirical fact: the higher population growth, the lower output per worker. High population growth corresponds with low living standards.
- $d\uparrow$

  $\Rightarrow$ ....................... rotates .................

  $\Rightarrow$ $k^*$....... , $y^*$ ............ , $c^*$...............

  $\Rightarrow$ In transition to the new growth path, $k^*$....... from $k_1^*$ to $k_2^*$, $y^*$ ............ , $c^*$............... (jump). Hence, all aggregate variables $(Y=y^*N, K=k*N, C=c^*N, S=sY^*)$ grows at rate .......

  $\Rightarrow$When the economy has arrived the new growth path $k^*$......................., therefore, $y^*, c^*$................... Hence, all aggregate variables $(Y=y^*N, K=k*N, C=c^*N, S=sY^*)$ grows at rate ......

  $\Rightarrow$Intuition: More of current savings is required just to keep the amount of capital per capita constant. In equilibrium output per capita and capital per capital decrease.