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2)

$$m = \frac{\delta u'(c_s^*)}{u'(c_0^*)} = \delta \frac{c_0^*}{c_s^*}$$

$$p_s = \pi_s m_s 1$$

$$p_s = \pi_s \cdot \delta \frac{c_0^*}{c_s^*} .$$

3) From the optimal intertemporal allocation of resource

$$\delta E[u'(c_s) R_s] = u'(c_0)$$

$$\frac{1}{R_s} = \delta E\left[\left(\frac{c_0}{c_s}\right)^{1-\gamma}\right]$$

$$R_s = \frac{1}{\delta}\left(\frac{c_s}{c_0}\right)^{1-\gamma}$$

$$\frac{\partial R_s}{\partial \frac{c_s}{c_0}} = \frac{1-\gamma}{\delta}\left(\frac{c_s}{c_0}\right)^{-\gamma}$$

$$\frac{\partial R_s}{\partial \frac{c_s}{c_0}} = (1-\gamma)\frac{R_s}{\frac{c_s}{c_0}}$$

$$\xi = \frac{R_s}{\frac{c_s}{c_0}} \cdot \frac{\partial \frac{c_1}{c_0}}{\partial R_s} = \frac{1}{1-\gamma}$$

4)

a) $p_s = p' X^{-1} e_s$

$$p = \begin{bmatrix} \frac{1}{1.05} \\ 6 \end{bmatrix} \qquad X = \begin{bmatrix} 1 & 10 \\ 1 & 5 \end{bmatrix} \qquad X^{-1} = \begin{bmatrix} -1 & 2 \\ 0.2 & -0.2 \end{bmatrix}$$

$$p_1 = \begin{bmatrix} \frac{1}{1.05} & 6 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0.2 & -0.2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad p_2 = \begin{bmatrix} \frac{1}{1.05} & 6 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0.2 & -0.2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \underline{0.2476} \qquad = \underline{0.7048}$$

risk neutral probabilities $\hat{\pi}_1 \equiv p_1 R_f = 0.2476(1.05) = 0.2600$

$\hat{\pi}_2 \equiv p_2 R_f = 0.7048(1.05) = 0.7400$

b)

$$m_1 = \frac{p_1}{\pi_1} = \frac{0.2476}{0.5} \qquad m_2 = \frac{p_2}{\pi_2} = \frac{0.7048}{0.5}$$

$$\underline{m_1 = 0.4952} \qquad \underline{m_2 = 1.4096}$$

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6)

a)

$$0 = E[m(R_i - R_f)]$$

$$= E[(a+bR_m)(R_i - R_f)]$$

$$= aE(R_i) - aR_f + bE(R_m R_i) - bR_f E(R_m)$$

$$0 = a(E(R_i) - R_f) + b[E(R_m)E(R_i) + cov(R_m, R_i) - R_f \cdot E(R_m)]$$

$$0 = (E(R_i) - R_f)(a + bE(R_m)) + b\,cov(R_m, R_i)$$

$$E(R_i - R_f) = \frac{-b\,cov(R_m, R_i)}{a + bE(R_m)}$$

$$E(R_i - R_f) = -\frac{cov(R_m, R_i)}{\sigma_m^2} \cdot \frac{b\sigma_m^2}{a + bE(R_m)}$$

$$E(R_i) - R_f = -\beta_i \cdot \frac{b\sigma_m^2}{a + bE(R_m)}$$

$$\beta_i \gamma = -\beta_i \cdot \frac{b\sigma_m^2}{a + bE(R_m)}$$

$$\gamma = -\frac{b\sigma_m^2}{a + bE(R_m)}$$

b) cont.

b) For risk-free asset,

$$\frac{1}{R_f} = E(m)$$

$$\frac{1}{R_f} = E(a+bR_m)$$

$$a = \frac{1}{R_f} - bE(R_m) \quad -(1)$$

For market portfolio,

$$1 = E[(a+bR_m)R_m]$$

$$1 = aE(R_m) + bE(R_m^2)$$

sub a from (1)

$$1 = \left(\frac{1}{R_f} - bE(R_m)\right)E(R_m) + bE(R_m^2)$$

$$1 = \frac{E(R_m)}{R_f} - b(E(R_m))^2 + bE(R_m^2)$$

$$1 = \frac{E(R_m)}{R_f} + b\sigma_m^2$$

$$b = \frac{1 - \frac{E(R_m)}{R_f}}{\sigma_m^2}$$

$$b = \frac{R_f - E(R_m)}{R_f \sigma_m^2}$$

$$a = \frac{1}{R_f} - \left[\frac{R_f - E(R_m)}{R_f \sigma_m^2}\right] E(R_m)$$

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c)

rewrite a as

$$a = \frac{\sigma_m^2 + E(R_m)\left(E(R_m) - R_f\right)}{R_f\,\sigma_m^2}$$

rewrite b as

$$b = -\frac{E(R_m) - R_f}{R_f\,\sigma_m^2}$$

$$a + bE(R_m) = \frac{\cancel{\sigma}^2_m + E\cancel{(R_m)(E(R_m) - R_f)} - E(R_m)\cancel{(E(R_m) - R_f)}}{R_f\,\cancel{\sigma}^2_{\cancel{m}}}$$

$$a + bE(R_m) = \frac{1}{R_f}$$

From (a)

$$\gamma = \frac{-b\sigma_m^2}{a + bE(R_m)}$$

$$= \frac{E(R_m) - R_f}{R_f\,\sigma_m^2}\,\sigma_m^2 R_f$$

$$\gamma = E(R_m) - R_f$$