Solution to Homework 5

CHAPTER 21

9. a. $uS_0 = 130 \Rightarrow P_u = 0$

$dS_0 = 80 \Rightarrow P_d = 30$

The hedge ratio is: $H = \frac{P_u - P_d}{uS_0 - dS_0} = \frac{0-30}{130-80} = -\frac{3}{5}$

b.

Riskless Portfolio	$S_T = 80$	$S_T = 130$
Buy 3 shares	240	390
Buy 5 puts	150	0
Total	390	390

Present value = $390/1.10 = $354.545

c. The portfolio cost is: 3S + 5P = 300 + 5P

The value of the portfolio is: $354.545

Therefore: 300 + 5P = $354.545 → P = $54.545/5 = $10.91

10. The hedge ratio for the call is: $H = \frac{C_u - C_d}{uS_0 - dS_0} = \frac{20-0}{130-80} = \frac{2}{5}$

Riskless Portfolio	S = 80	S = 130
Buy 2 shares	160	260
Write 5 calls	0	-100
Total	160	160

Present value = $160/1.10 = $145.455

The portfolio cost is: 2S – 5C = $200 – 5C

The value of the portfolio is: $145.455

Therefore: C = $54.545/5 = $10.91

Does P = C + PV(X) – S?

10.91 = 10.91 + 110/1.10 – 100 = 10.91

11. $d_1 = 0.2192 \Rightarrow N(d_1) = 0.5868$

$d_2 = -0.1344 \Rightarrow N(d_2) = 0.4465$

$Xe^{-rT} = 49.2556$

C = \$50 × 0.5868 – 49.2556 × 0.4465 = \$7.34

12. P = \$6.60

This value is derived from our Black-Scholes spreadsheet, but note that we could have derived the value from put-call parity:

$P = C + PV(X) - S_0$ = \$7.34 + \$49.26 – \$50 = \$6.60

18. The best estimate for the change in price of the option is:

Change in asset price × delta = −\$6 × (-0.65) = \$3.90

23. Implied volatility has increased. If not, the call price would have fallen as a result of the decrease in stock price.

24. Implied volatility has increased. If not, the put price would have fallen as a result of the decreased time to expiration.

CHAPTER 22

8. a. $F_0 = S_0(1 + r_f)$ = \$150 × 1.03 = \$154.50

b. $F_0 = S_0(1 + r_f)^3$ = \$150 × 1.03^3 = \$163.91

c. F_0 = 150 × 1.06^3 = \$178.65

11. The put-call parity relation states that: But spot-futures parity tells us that:

$$C = P + S_0 - \frac{X}{(1+r_f)^T} \qquad\qquad F = S_0 \times (1+r_f)^T$$

Substituting, we find that:

$$P = C - S_0 + \frac{[S_0 \times (1+r_f)^T]}{(1+r_f)^T} = C - S_0 + S_0 = C$$

16. The parity value of F is: 1,300 × (1 + 0.04 – 0.01) = 1,339

The actual futures price is 1,330, too low by 9.

Arbitrage Portfolio	CF now	CF in 1 year
Short Index	1,300	$-S_T - (0.01 \times 1{,}300)$
Buy Futures	0	$S_T - 1{,}330$
Lend	−1,300	1,300 × 1.04

18. a. The current yield for Treasury bonds (coupon divided by price) plays the role of the dividend yield.