4 Testing Hypotheses about a Single Linear Combination of the Parameter

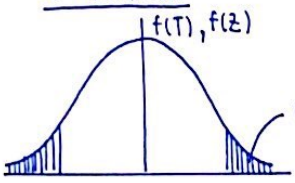
Consider

$$\log(wage) = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 \exp er + u$$

where jc = number of years attending a two-year college (Junior college 의미)
$univ$ = number of years at a four-year college
$\exp er$ = months in the workforce.
We want to test whether $\beta_1 = \beta_2$. → Test if the returns from 1 more year of education at a Junior college is the same as that of the university

$H_0: \beta_1 = \beta_2 \rightarrow H_0: \beta_1 - \beta_2 = 0$
against
$H_a: \beta_1 \neq \beta_2 \rightarrow H_a: \beta_1 - \beta_2 \neq 0$

2-tailed test

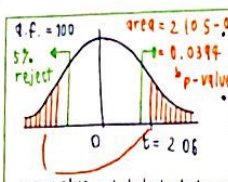


$$t = \frac{(\hat{\beta}_1 - \hat{\beta}_2) - 0}{s.e.(\hat{\beta}_1 - \hat{\beta}_2)}$$ → we compute this t-statistic and compare with the critical value

where $s.e.(\hat{\beta}_1 - \hat{\beta}_2) = \sqrt{var(\hat{\beta}_1 - \hat{\beta}_2)}$

$$= \sqrt{var(\hat{\beta}_1) + var(\hat{\beta}_2) - 2\,cov(\hat{\beta}_1, \hat{\beta}_2)}$$

not very straight forward to calculate
↳ we use a variable transformation trick



- Suppose we calculate a t-statistic = 2.06
- Suppose we are testing $H_0: \beta_j = 0$, $H_a: \beta_j \neq 0$
  ↳ 2-tailed test
- p-value = total shaded area
  ↳ = significant level which we will reject the H_0 or Prob that we will reject H_0

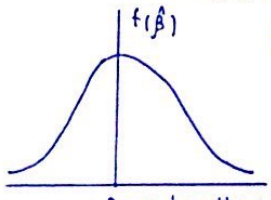
★ p-value < significance level
↳ reject H_0

Inference → Hypothesis testing about β (true parameter)

$wage = \beta_0 + \beta_1 educ + \beta_2 experience + \ldots + u$

We want to test hypothesis about the true impact (β) of each X variables (educ, experience) on the dependent variable (Y)

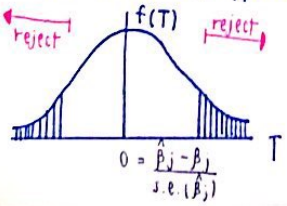
BUT we don't know what the true β are. So, we use $\hat{\beta}$ (estimator) and $s.e.(\hat{\beta})$ to test the hypothesis



1) Test if β = some number
e.g. $\beta_j = 0$ → X_j has no impact on Y
$\beta_j = 1$ → 1 unit↑ in X_j correspond to 1 unit↑ in Y

→ t-test
$\frac{\hat{\beta}_j - \beta_j}{s.e.(\hat{\beta}_j)} \sim t_{d.f.}$

significant level = total area in the rejection region

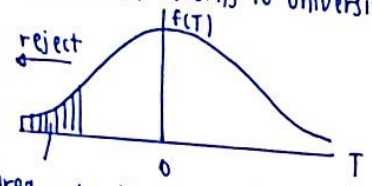


another possible hypothesis test (one-tailed alternative)

$H_0: \beta_1 = \beta_2 \rightarrow H_0: \beta_1 - \beta_2 = 0$

$H_a: \beta_1 < \beta_2 \rightarrow H_a: \beta_1 - \beta_2 < 0$

• It is assumed that β_1 would not be more than β_2 (returns to 2-year college would never be more than returns to university education)



area = sig. level

$$t = \frac{(\hat{\beta}_1 - \hat{\beta}_2) - 0}{s.e.(\hat{\beta}_1 - \hat{\beta}_2)}$$

Consider the multiple regression model, assume MLR 1 - 6 are satisfied

$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + u$

You would like to test the $H_0: \beta_1 - 3\beta_2 = 1$ → H_a: otherwise is true

① Write the t-statistic for testing H_0

$$t = \frac{(\hat{\beta}_1 - 3\hat{\beta}_2) - 1}{s.e.(\hat{\beta}_1 - 3\hat{\beta}_2)}$$

② Define $\theta_1 = \hat{\beta}_1 - 3\hat{\beta}_2$

$H_0: \theta_1 = 1$ $\quad t = \frac{\hat{\theta}_1 - 1}{s.e.(\hat{\theta}_1)}$ → we need our regression to have θ_1 in it. So, STATA or OLS estimation will automatically give $\hat{\theta}_1$ and $s.e.\hat{\theta}_1$

$H_a: \theta_1 \neq 1$

Now, $\hat{\beta}_1 = \theta_1 + 3\hat{\beta}_2$ or $\beta_1 = \theta_1 + 3\beta_2$

sub in the main regression and get

$Y = \beta_0 + (\theta_1 + 3\beta_2)X_1 + \beta_2 X_2 + \beta_3 X_3 + u$

$Y = \beta_0 + \theta_1 X_1 + \beta_2 (X_2 + 3X_1) + \beta_3 X_3 + u$

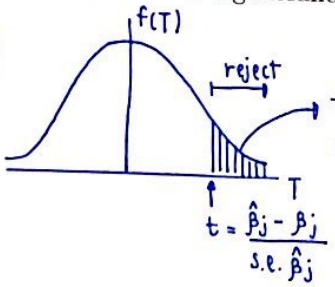
Now, the explanatory variables are going to be $X_1, X_2 + 3X_1$ and X_3

• We can calculate $t = \frac{\hat{\theta}_1 - 1}{s.e.\hat{\theta}_1}$

5 Computing p-Values for t-Tests

• What is the significance level given the computed t-statistics?

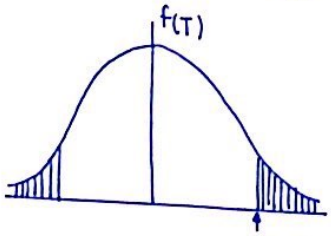
1-tailed



This shaded area in the rejection region is the p-value

$t = \frac{\hat{\beta}_j - \beta_j}{s.e.\hat{\beta}_j}$

2-tailed



t = computed $t = \frac{\hat{\beta}_j - \beta_j}{s.e.\hat{\beta}_j}$

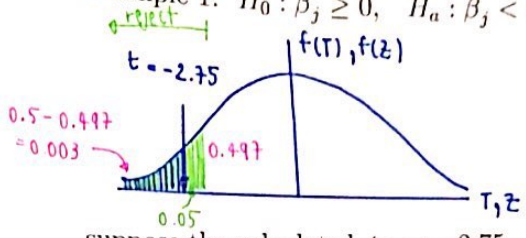
• p-value : $P(|T| > |t|)$

T = t-distributed random variable with d.f. = n-k-1

t = computed t-statistic

→ p-value = probability that a random T value will be greater (in the | | term) than our t in the Hypothesis test.

Example 1: $H_0 : \beta_j \geq 0, \quad H_a : \beta_j < 0, \quad$ d.f.$= 140$. → z-table



→ p-value = what should be the significant level, given the critical value of -2.75
↳ find the shaded area

suppose the calculated $t_{\hat{\beta}_j} = -2.75$

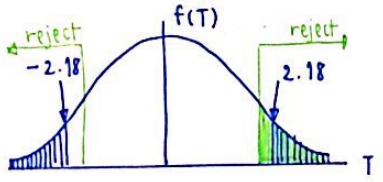
↳ $t_{\hat{\beta}_j} = \dfrac{\hat{\beta}_j - \beta_j}{s.e.(\hat{\beta}_j)}$

- From the z-table, the value -2.75 corresponds to area = 0.003
- Thus, p-value = 0.003
- Would we reject H_0 if we use the significance level = 5%? Yes.

  Rule : we reject H_0 if p-value < sig level

Example 2: $H_0 : \beta_j = a_j, \quad H_a : \beta_j \neq a_j, \quad$ d.f.$= 18$.

↳ use t-table



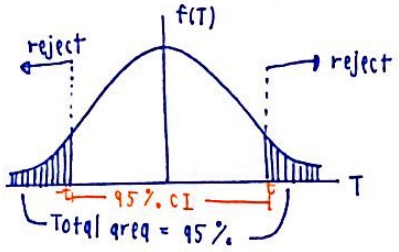
suppose the calculated $t_{\hat{\beta}_j} = -2.18$

- From the t-table, the value -2.18 corresponds to area = 0.02 to 0.05
- Thus, p-value = is between 0.02 and 0.05.
- Would we reject H_0 if we use the significance level = 5%? Yes, reject H_0 because the area is less than 0.05 or p-value < 0.05

6 Confidence Intervals (CI)

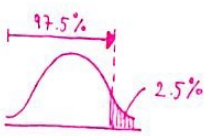
- Confidence Intervales for the POPULATION PARAMETER (β_j)

  → The range of values that would capture the true β_j at a 95% chance
- A 95% CI of β_j is given by

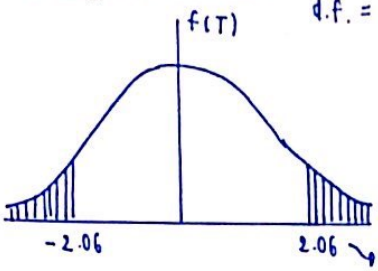


CI ⇒ $\hat{\beta}_j \pm c \times s.e.(\hat{\beta}_j)$

c is the 97.5 percentile in the t-distribution with n-k-1 d.f.



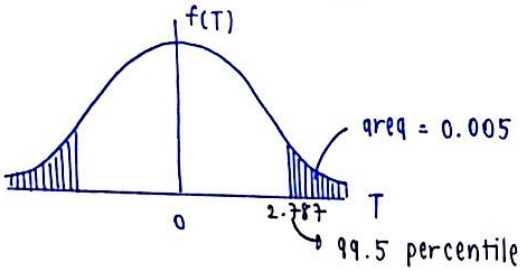
Example 1: **95% CI**



→ value of 97.5 percentile in the t_{25} distribution

The 95% CI for $\hat{\beta}_j = \left[\hat{\beta}_j - 2.06\, s.e.(\hat{\beta}_j),\ \hat{\beta}_j + 2.06 \cdot s.e.(\hat{\beta}_j)\right]$

Example 2: **99% CI** d.f. = 25



→ 99.5 percentile or area = 1 - 0.005 = 0.995

The 99% CI for $\hat{\beta}_j = \left[\hat{\beta}_j - 2.787\, s.e.(\hat{\beta}_j),\ \hat{\beta}_j + 2.787\, s.e.(\hat{\beta}_j)\right]$

F-test motivation

→ We want to test the significance of a group of hypotheses (multiple hypotheses)

$Grade_{325} = \beta_0 + \beta_1 \#times\_front + \beta_2 \#times\_back + \beta_3 hr\_study + \beta_4 past\_GPA + \beta_5 gender + u$

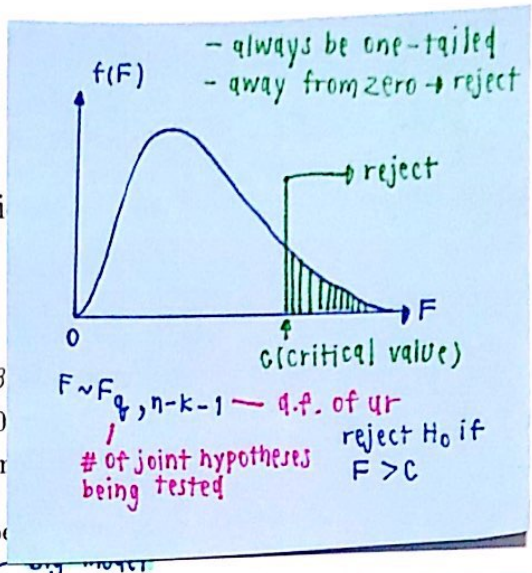
H_0 : seat position doesn't have impact on GPA → $\beta_1 = 0$ and $\beta_2 = 0$ → $\beta_1 = \beta_2 = 0$

H_a : seat position matters → $\beta_1 \neq 0$ and $\beta_2 \neq 0$ + $\beta_1 \neq 0$ or $\beta_2 \neq 0$ → at least one of the β_1, β_2 not equal to 0

7 Testing Multiple Linear Restri

Suppose the model is specified by

$y = \beta_0 + \beta$
$H_0 : \beta_2 = 0$
$H_1 : H_0$ is r



x_2 both

We can use the F-test to test this typ

1. Our full model is called the "unrestricted" model (ur). Suppose it can be expressed as: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + u$ → is true → Reject H_0

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \ldots + \beta_k x_k + u$$

2. The model which takes out x (which we think its associated $\beta = 0$) is called the restricted model (r). ← small model

$Y = \beta_0 + \beta_1 X_1 + u$ is true → donot reject H_0

Suppose there are q number of β that we should like to perform a joint-test of $=0$.
e.g. in this model $q = 2$

$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_{k-q} X_{k-q} + u$

$H_0 : \beta_{k-q+1} = \beta_{k-q+2} = \ldots = \beta_k = 0$
(the last q $\beta_s = 0$)
H_a : H_0 is not true

$$Y = \underbrace{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_{k-q} X_{k-q}}_{(r)} + \underbrace{\beta_{k-q+1} X_{k-q+1} + \beta_{k-q+2} X_{k-q+2} + \ldots + \beta_k X_k + u}_{ur}$$

$$F \equiv \frac{\frac{SSR_r - SSR_{ur}}{q}}{\frac{SSR_{UR}}{(n-k-1)}}$$

This is always ⊕ b/c $SSR_{ur} < SSR_r$
(Every time you add one more X, the model will be better explained)

d.f. of the ur model

- So, if every time you add 1 more X variable, the SSR↓ and R^2↑, why don't we just keep the additional X in the model?
→ Because every time we add 1 more X, $Var(\hat{\beta}_s)$ will increase, making the prediction of β less precise. So, we only keep the additional X, if it/they can improve the model enough.

can↓ SSR (↑R^2) enough
can significantly ↓ SSR and ↑R^2

3. Some useful facts

① $R^2_{ur} > R^2_r$ because any additional X would increase R^2 (improve fit)
$SSR_{ur} < SSR_r$

② By including more X, the model is certainly better explained. However, we would like to reject H_0 if the inclusion of extra variables does not improve the model enough

4. Other ways to calculate the F-statistics:

From $R^2 = 1 - \frac{SSR}{SST}$

We have $F \equiv \left(\frac{R^2_{ur} - R^2_r}{q} \right) \Big/ \frac{1 - R^2_{ur}}{n-K-1}$

q: # of β that are set to 0

$n-K-1$: # of obs − # of slope β − intercept

If we want to test the overall significance of the model

$H_0: \beta_1 = \beta_2 = \beta_3 = \ldots = \beta_K = 0$, H_a: otherwise

$F \equiv \frac{R^2/K}{(1-R^2)/(n-K-1)}$ → R^2 of ur → r model has no X at all

Example: Suppose we are interested in understanding the determinant of a baseball player's salary.

Y *salary* = season salary

r { ur { *years* = years in major leagues
gamesyr = games per year in the league
bavg = career batting average
hrunsyr = homeruns per year
rbisyr = runs batted in per year

If we want to test whether performance has any impact on salary

$H_0: \beta_{bavg} = \beta_{hrunsyr} = \beta_{rbisyr} = 0$

H_a: otherwise is true

- the unrestricted model (ur) is defined by

"ur model"

```
. regress log_salary years gamesyr bavg hrunsyr rbisyr
```

Source	SS	df	MS
Model	308.989208	5	61.7978416
Residual	183.186327	347	.527914487
Total	492.175535	352	1.39822595

Number of obs	= 353
F(5, 347)	= 117.06
Prob > F	= 0.0000
R-squared	= 0.6278
Adj R-squared	= 0.6224
Root MSE	= .72658

log_salary	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
years	.0688626	.0121145	5.68	0.000	.0450355	.0926898
gamesyr	.0125521	.0026468	4.74	0.000	.0073464	.0177578
bavg	.0009786	.0011035	0.89	0.376	-.0011918	.003149
hrunsyr	.0144295	.016057	0.90	0.369	-.0171518	.0460107
rbisyr	.0107657	.007175	1.50	0.134	-.0033462	.0248776
_cons	11.19242	.2888229	38.75	0.000	10.62435	11.76048

may not have individually impact but must be three factors combined to create impact on salary

"Use F-test" to find whether three factors can jointly impact or not

When considering each of the performance X one-by-one, none of them has significant impact at 5%

- the restricted model (r) is defined by

"r model"

```
. regress log_salary years gamesyr
```

	Source	SS	df	MS
SSE	Model	293.864058	2	146.932029
SSR	Residual	198.311477	350	.566604221
SST	Total	492.175535	352	1.39822595

Number of obs	= 353
F(2, 350)	= 259.32
Prob > F	= 0.0000
R-squared	= 0.5971
Adj R-squared	= 0.5948
Root MSE	= .75273

log_salary	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
years	.071318	.012505	5.70	0.000	.0467236	.0959124
gamesyr	.0201745	.0013429	15.02	0.000	.0175334	.0228156
_cons	11.2238	.108312	103.62	0.000	11.01078	11.43683

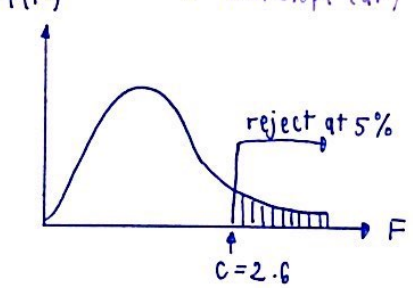
But when performing an F-test, performances have joint impact.

Now, our H_0 and H_a becomes

$$F \equiv \frac{\left(\frac{SSR_r - SSR_{ur}}{q}\right)}{\left(\frac{SSR_{ur}}{n-k-1}\right)} \equiv \frac{\frac{198.311 - 183.186}{3}}{\frac{183.186}{353-5-1}} \approx 9.55$$

n: # of obs; k: # of slope (ur); q: allow 5% mistake

Let's use 5% level of significance



Since $F = 9.55 > 2.6$, we reject H_0 at 5% level and conclude that performances have joint effects on salary.

F table page 966 $F_{3,\infty}$

$Pr = 0.05$, $df\ N_1 = 3$, $df\ N_2 = \infty$

8 How the Hypothesis Testing is done in Practice

1. Check the values of $t-statistic$ reported by the statistical software (i.e. STATA, SPSS, SAS)

   $\Rightarrow$ These $t-statistics$ are to test $H_0 : \beta_i = 0$

   $\Rightarrow$ If the d.f. > 30, then when $t > 1.96$, we can reject H_0 with 5% significant level (2-table)

   $\Rightarrow$ **When $t > 1.96$**, we can say that β_i is **statistically significant** at 5% level. (value of $\beta_i \neq 0$)

   $\Rightarrow$ **When $t < 1.96$** we can say that β_i is **not statistically significant** at 5% level.

   $\Rightarrow$ If $t < 1.96$ we can drop x_i from the model

   $\Rightarrow$ After we drop x_i, we estimate the new regression function and obtain a new set of $\hat{\beta}$.

2. We can also perform other hypothesis testings of interest.

   e.g. $H_0 : \beta_i = \beta_j$

   or $H_0 : \beta_i = 5$ etc.

   or perform an F-test for testing multiple linear restrictions

3. Usually, in economics, the estimation results are reported using this form

Dependent Variable: log(*salary*)			
Independent Variables	**(1)**	**(2)**	**(3)**
log(*sales*) (sales)	.224 (.027)	.158 (.040)	.188 (.040)
log(*mktval*) (other company performance)	—	.112 (.050)	.100 (.049)
profmarg (other company performance)	—	−.0023 (.0022)	−.0022 (.0021)
ceoten (CEO characteristics)	—	—	.0171 (.0055)
comten (CEO characteristics)	—	—	−.0092 (.0033)
intercept	4.94 (0.20)	4.62 0.25)	4.57 (0.25)
Observations *R*-squared	177 .281	177 .304	177 .353

Simple regression with 1 X

Multiple Regression Analysis : Further Issues

1 Data scaling on OLS statistics

When we change the unit of measurment of a variable, the value of estimators would chance accordingly. For example

$$\widehat{bweght} = \widehat{\beta}_0 + \widehat{\beta}_1 cigs + \widehat{\beta}_2 fa\min c,$$

↳ in gram

where
$bwght$ = child birth weight, in grams.
$cigs$ = number of cigarettes smoked by the mother while pregnant, per day.
$fa\min c$ = annual family income, in thousands of dollars.

- What if we use bwght in kilograms ??

1 kg = 1000 g

$$\widehat{bweght}_{kg} = \frac{\widehat{bweght}_g}{1000} = \frac{\hat{\beta}_0}{1000} + \frac{\hat{\beta}_1 cigs}{1000} + \frac{\hat{\beta}_2 faminc}{1000}$$

$$\widehat{bweght}_{kg} = \hat{\alpha}_0 + \hat{\alpha}_1 cigs + \hat{\alpha}_2 faminc$$

```
gen bwght_g  = bwght * 28.35
gen bwght_kg = bwght_g / 1000
```
} generate new variable

- What if we use faminc in USD (instead of 1000 USD)

$$bweght_g = \hat{\beta}_0 + \hat{\beta}_1 cigs + \frac{\hat{\beta}_2}{1000} faminc_{USD}$$

← The value of this variable is going to be 1000 times larger than faminc

$$= \hat{\beta}_0 + \hat{\beta}_1 cigs + \hat{\theta}_2 faminc_{USD}$$

$$\hat{\theta}_2 = \frac{\hat{\beta}_2}{1000}$$

in other words $\hat{\theta}_2$ = impact of 1 USD ↑ in income

$\hat{\beta}_2$ = impact of 1000 USD ↑ in income

```
gen faminc_usd = faminc * 1000
```

- What if we use bweght in kg & income in THB

$$bweght_{kg} = \frac{\hat{\beta}_0}{1000} + \frac{\hat{\beta}_1}{1000} cigs + \frac{\hat{\beta}_2}{1000} faminc_{THB}$$

30,000

← This value is going to be 30,000 times larger than faminc (assume 30 THB = 1 USD)

2 More on functional forms

- Logarithmic Functional Form

$\frac{d \ln x}{dx} = \frac{1}{x} \rightarrow d\ln x = \frac{dx}{x}$

usually means natural log in econometrics

$$\log(y) = \beta_0 + \beta_1 \log(x_1) + \beta_2 x_2 + u$$

$\Delta y = y_1 - y_2$, $\Delta x_1 = x_{11} - x_{12}$

$$\beta_1 = \frac{d \log(y)}{d \log(x_1)} = \frac{\frac{1}{y} dy}{\frac{1}{x_1} dx_1} = \frac{\frac{1}{y} \Delta y}{\frac{1}{x_1} \Delta x_1} = \frac{100 \frac{1}{y} \Delta y}{100 \frac{1}{x_1} \Delta x_1} = \frac{\% \Delta y}{\% \Delta x_1}$$

with the log y and log x format, the coefficient is going to be the elasticity (x_1 elasticity of y)
(price) (demand)

$$\beta_2 = \frac{d \log(y)}{d x_2} = \frac{\frac{1}{y} dy}{dx_2} = \frac{\frac{1}{y} \Delta y}{\Delta x_2}$$

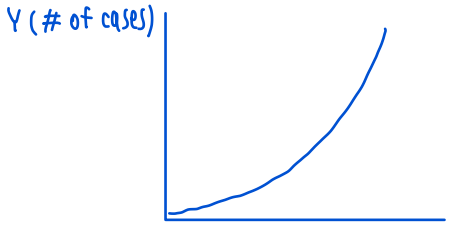
if we want the upper term to be % change, then

$$100 \beta_2 = \frac{100 \frac{1}{y} \Delta y}{\Delta x_2}$$

$100 \beta_2 = \frac{\% \Delta y}{\Delta x_2} \rightarrow 100 \beta_2 = \%$ Δ in y given that x_2 increases by 1 unit.

- Models with Quadratics → squares
  capture increasing/decreasing marginal effects (slope of the relationship between X & Y is not constant

COVID-19 example

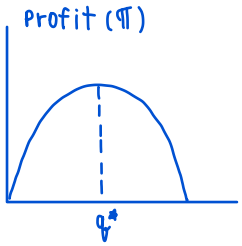


$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + u$

$\frac{dy}{dx} = \beta_1 + 2\beta_2 X$

(+) (+) days

Decreasing returns



$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + u$

$\frac{dy}{dx} = \beta_1 + 2\beta_2 X$

(+) (−)

$\pi = (P - MC) q$ ← Assume mc = 10

Demand: $P = 100 - q$

$\pi = (100 - q - 10) q$

β_1 is positive

FOC $\frac{\partial \pi}{\partial q} = 0 = 90 - 2q$

β_2 is negative

same result

Example : Effects of Pullution on Housing Prices

$$\log(price) = \beta_0 + \beta_1 \log(nox) + \beta_2 \log(dist) + \beta_3 rooms + \beta_4 room^2 + \beta_5 stratio + u$$

where
price = housing price
nox = level of pollution
dist = distance from downtown
rooms = number of rooms
stratio = average student per teacher ratio
The estimation result is given by

In the US or many countries, students can apply to schools in the area without having to take any test. So, the lower stratio, the better the school.

```
regress lprice lnox dist rooms rooms_sq stratio
```

Source	SS	df	MS
Model	51.4933152	5	10.298663
Residual	33.0889098	500	.06617782
Total	84.582225	505	.167489554

Number of obs	= 506
F(5, 500)	= 155.62
Prob > F	= 0.0000
R-squared	= 0.6088
Adj R-squared	= 0.6049
Root MSE	= .25725

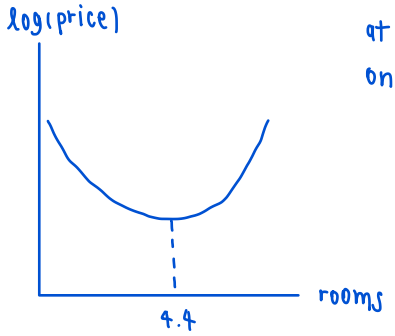
log(price)
log(nox)

lprice	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnox	β_1 -.9767545	.0995938	-9.81	0.000	-1.172429	-.7810806
dist	β_2 -.0321972	.0094013	-3.42	0.001	-.050668	-.0137264
rooms	β_3 -.5528032	.1612965	-3.43	0.001	-.8697056	-.2359007
rooms_sq	β_4 .0624697	.0124867	5.00	0.000	.0379368	.0870025
stratio	β_5 -.0486667	.0058131	-8.37	0.000	-.0600879	-.0372455
_cons	13.59154	.5650901	24.05	0.000	12.4813	14.70178

|t| > 1.96 for all variables
↳ All variables are significant
all < 0.05

Consider the effect of "room"

$$\frac{d\log(price)}{d\,rooms} = \beta_3 + 2\beta_4\,rooms = -0.553 + 2(0.062)\,rooms$$



at how many rooms does 1 additional room has a positive impact on log(price)??

$0 = -0.553 + 2(0.062)\cdot rooms$

$rooms = 4.4$

At 4.4 rooms or more
↳ round up to 5 rooms or more

What wold be the % change in price when the number of room increases from 5 to 6?

$$\frac{d\log(price)}{d\,rooms} = -0.553 + 2(0.062)\,rooms$$

$$\frac{100\,\frac{1}{price}\,d\,price}{d\,rooms} = 100\left(-0.553 + 2(0.062)(5)\right) = 100(0.067) = 6.7\%\ \text{increase}$$

what about % Δ in price when # rooms increases from 5 to 7?

% Δ price $= 100(-0.553 + 2(0.062)(6)) = 19.1\%$

total % Δ in price when # rooms ↑ from 5 to 7 is $6.7 + 19.1 = 25.8\%$

3 Models with Interaction Terms

→ used when the impact of one variable depends on the value (level) of another variable

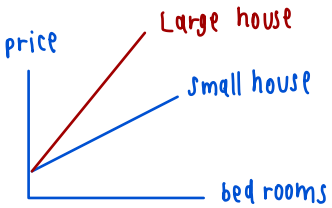
Consider

$$price = \beta_0 + \beta_1 sqrft + \beta_2 bdrms + \beta_3 \underbrace{sqrft \times bdrms}_{x_3} + \beta_4 bthrms + u$$

x_1 x_2 x_1 x_2 x_4

where

$price$ = housing price
$sqrft$ = house size (square feet)
$bdrms$ = number of bedrooms
$bthrms$ = number of bathrooms



$\frac{\Delta price}{\Delta bdrms} = \beta_2 + \beta_3 sqrft$

→ if $\beta_2 > 0$ then, an additional bedroom would increase price more for a larger house

4 More on the Goodness-of-Fit and Selection of Regressors

- Adding more regressors ALWAYS improve fit → R^2 always increase

→ But we lose the degree of freedom (d.f. = free data point used to estimate the parameter)

→ 1 data point is sacrificed every time we estimate a parameter.

→ using R^2 would not punish "having too many regressors"

→ We use adjusted-R^2 or $\bar{R}^2$ when we want to punish adding too many regressors

$$R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{SSR/n}{SST/n}$$

$$\text{adj. } R^2 = 1 - \frac{SSR/(n-k-1)}{SST/(n-1)}$$

→ If we have more k, d.f. = n-k-1 ↓, SSR/(n-k-1) ↑, adj. R^2 ↓

Using adjusted R-squared to choose between non-nested models (one model is not a subset of another).

Consider Model 1

$$\begin{aligned} \widehat{salary} &= \underset{(223.90)}{830.63} + \underset{(0.0089)}{0.0163}sales + \underset{(11.08)}{19.63}roe \\ n &= 209, \quad R^2 = 0.029, \quad \bar{R}^2 = 0.020 \end{aligned}$$

Consider Model 2

$$\begin{aligned} \widehat{\log(salary)} &= \underset{(0.29)}{4.36} + \underset{(0.033)}{0.2751}\log(sales) + \underset{(0.004)}{0.0179}roe \\ n &= 209, \quad R^2 = 0.282, \quad \bar{R}^2 = 0.275 \end{aligned}$$

Multiple Regression Analysis with Qualitative Information:

1 Outline

- Describing qualitative information
- Using a single dummy independent variable
- Using dummy variables for multiple categories
- Interactions involving dummy variables
- A binary dependent variable (Y variable): The linear probability model

2 Describing Qualitative Information

- "Female" and "Married" are qualitative variable.
- We arbitarily assign a dummy variable to decribe them.

$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$

$$married = \begin{cases} 1 & \text{if married} \\ 0 & \text{otherwise (of if single)} \end{cases}$$

TABLE 7.1

A Partial Listing of the Data in WAGE1.RAW

person	wage	educ	exper	female	married
1	3.10	11	2	1	0
2	3.24	12	22	1	1
3	3.00	11	2	0	0
4	6.00	8	44	0	1
5	5.30	12	7	0	1
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
525	11.56	16	5	0	1
526	3.50	14	5	1	0