

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$

Use OLS to find the estimator of β_1 and β_2 . Interpret the regression.

1.2 Find \hat{Y}_i and \hat{u}_i and show that $\sum_{i=1}^n \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1)$, and $var(\hat{\beta}_2)$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of \hat{Y}_i and \hat{u}_i . Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass (\bar{X}, \bar{Y}) ?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i), var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

“Practice makes Perfect.”

(1.)

$$\bar{x} = \frac{63+72+78+81+87+75+75+90}{8} = \frac{621}{8} = 77.625$$

$$\bar{y} = \frac{2.8+3.4+3.0+3.5+3.6+3.0+2.7+3.7}{8} = \frac{25.7}{8} = 3.2125$$

$$\sum_{i=1}^n x_i^2 = 63^2 + 72^2 + 78^2 + 81^2 + 87^2 + 75^2 + 75^2 + 90^2 = 48717$$

$$\begin{aligned} \sum_{i=1}^n x_i y_i &= 63(2.8) + 72(3.4) + 78(3.0) + 81(3.5) + 87(3.6) + 75(3.0) + 75(2.7) + 90(3.7) \\ &= 2012.4 \end{aligned}$$

$$\sum_{i=1}^n x_i = 63+72+78+81+87+75+75+90 = 621$$

$$\sum_{i=1}^n y_i = 2.8+3.4+3.0+3.5+3.6+3.0+2.7+3.7 = 25.7$$

(1.1)

from

$$\hat{\beta}_1 = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}$$

$$= \frac{8(2012.4) - 621(25.7)}{8(48717) - (621)^2}$$

$$= \frac{16099.2 - 15959.7}{389736 - 385641}$$

$$= \frac{139.5}{4095} = 0.0341 \quad \times$$

from

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= 3.2125 - 0.0341(77.625)$$

$$= 0.5655 \quad \times$$

1.2

from $\hat{Y}_i = \beta_0 + \hat{\beta}_1 X_i$ where $\beta_0 = 0.5655$, $\beta_1 = 0.0741$

$$i=1, \hat{Y}_1 = 0.5655 + 0.0741(63) = 2.7128$$

$$i=2, \hat{Y}_2 = 0.5655 + 0.0741(72) = 3.0207$$

$$i=3, \hat{Y}_3 = 0.5655 + 0.0741(76) = 3.2253$$

$$i=4, \hat{Y}_4 = 0.5655 + 0.0741(81) = 3.3276$$

$$i=5, \hat{Y}_5 = 0.5655 + 0.0741(87) = 3.5322$$

$$i=6, \hat{Y}_6 = 0.5655 + 0.0741(75) = 3.123$$

$$i=7, \hat{Y}_7 = 0.5655 + 0.0741(75) = 3.123$$

$$i=8, \hat{Y}_8 = 0.5655 + 0.0741(90) = 3.6345$$

from $\hat{u}_i = Y_i - \hat{Y}_i$

$$i=1, \hat{u}_1 = 2.8 - 2.7128 = 0.0862$$

$$i=2, \hat{u}_2 = 3.4 - 3.0207 = 0.3793$$

$$i=3, \hat{u}_3 = 3.0 - 3.2253 = -0.2253$$

$$i=4, \hat{u}_4 = 3.5 - 3.3276 = 0.1724$$

$$i=5, \hat{u}_5 = 3.6 - 3.5322 = 0.0678$$

$$i=6, \hat{u}_6 = 3.0 - 3.123 = -0.123$$

$$i=7, \hat{u}_7 = 2.7 - 3.123 = -0.423$$

$$i=8, \hat{u}_8 = 3.7 - 3.6345 = 0.0655$$

$$\sum_{i=1}^n \hat{u}_i = 0.0862 + 0.3793 - 0.2253 + 0.1724 + 0.0678 - 0.123 - 0.423 + 0.0655$$

$$= -0.0001 \approx 0$$

(1.3)

$$\text{from var } (\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2}$$

$$= \frac{0.0862^2 + 3793^2 + (-0.1253)^2 + 0.1714^2 + 0.0674^2 + (-0.123)^2 + (-0.423)^2 + 0.0655^2}{8-2}$$

$$= \frac{0.4347}{6} = 0.0725 \quad \#$$

$$\text{from var } (\hat{\beta}_0) = \frac{\sum X_i^2}{n \sum X_i^2} \sigma^2$$

$$= \frac{48717}{8(514.3795)} (0.725)$$

$$= \frac{3531.9425}{4115.036} = 0.8583 \quad \#$$

$$\text{from var } (\hat{\beta}_1) = \frac{\sigma^2}{\sum X_i^2} = \frac{\sigma^2}{\sum (X_i - \bar{X})^2}$$

$$= \frac{0.0725}{[(63 - 77.625)^2 + (72 - 77.625)^2 + (78 - 77.625)^2 + (81 - 77.625)^2 + (87 - 77.625)^2 + (95 - 77.625)^2 + (15 - 77.625)^2 + (90 - 77.625)^2]}$$

$$= \frac{0.0725}{514.3795} = 0.00014 \quad \#$$

(2.)

$$\bar{x} = \frac{10+12+14+16+18+22+24+26+28+30}{10} = 20$$

$$\bar{y} = \frac{0+1+5+6+7+10+10+15+16+20}{10} = 9.1$$

$$\sum x_i^2 = 10^2 + 12^2 + 14^2 + 16^2 + 18^2 + 22^2 + 24^2 + 26^2 + 28^2 + 30^2 = 4440$$

$$\begin{aligned} \sum x_i y_i &= 10(0) + 12(1) + 14(5) + 16(6) + 18(7) + 22(10) + 24(10) + 26(15) + 28(16) + 30(20) \\ &= 1214 \end{aligned}$$

$$\sum x_i = 10+12+14+16+18+22+24+26+28+30 = 200$$

$$\sum y_i = 0+1+5+6+7+10+10+15+16+20 = 91$$

(2.1)

$$\text{from } \hat{\beta}_1 = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

$$= \frac{10(1214) - (200)(91)}{10(4440) - 200^2}$$

$$= \frac{3940}{4400} = 0.8955 \quad \# \rightarrow \text{slope of regression line}$$

$$\text{from } \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= 9.1 - 0.8955(20) = -8.81 \quad \# \rightarrow \text{y-intercept of regression line, } \hat{y}_0 = -8.81$$

2.2

from $\hat{Y}_i = \beta_1 + \hat{\beta}_2 x_i$ where $\beta_1 = -8.81$, $\beta_2 = 0.9955$

$i=1,$	\hat{Y}_1	$=$	$-8.81 + 0.9955(10)$	$=$	0.145
$i=2,$	\hat{Y}_2	$=$	$-8.81 + 0.9955(12)$	$=$	1.936
$i=3,$	\hat{Y}_3	$=$	$-8.81 + 0.9955(14)$	$=$	3.727
$i=4,$	\hat{Y}_4	$=$	$-8.81 + 0.9955(16)$	$=$	5.518
$i=5,$	\hat{Y}_5	$=$	$-8.81 + 0.9955(18)$	$=$	7.309
$i=6,$	\hat{Y}_6	$=$	$-8.81 + 0.9955(20)$	$=$	10.591
$i=7,$	\hat{Y}_7	$=$	$-8.81 + 0.9955(24)$	$=$	12.682
$i=8,$	\hat{Y}_8	$=$	$-8.81 + 0.9955(26)$	$=$	14.473
$i=9,$	\hat{Y}_9	$=$	$-8.81 + 0.9955(28)$	$=$	16.264
$i=10,$	\hat{Y}_{10}	$=$	$-8.81 + 0.9955(30)$	$=$	18.055

~~✗~~

from $\hat{u}_i = Y_i - \hat{Y}_i$

$i=1,$	\hat{u}_1	$=$	$0 - 0.145$	$=$	-0.145
$i=2,$	\hat{u}_2	$=$	$2 - 1.936$	$=$	0.064
$i=3,$	\hat{u}_3	$=$	$5 - 3.727$	$=$	1.273
$i=4,$	\hat{u}_4	$=$	$6 - 5.518$	$=$	0.482
$i=5,$	\hat{u}_5	$=$	$7 - 7.309$	$=$	-0.309
$i=6,$	\hat{u}_6	$=$	$10 - 10.591$	$=$	-0.591
$i=7,$	\hat{u}_7	$=$	$10 - 12.682$	$=$	-2.682
$i=8,$	\hat{u}_8	$=$	$15 - 14.473$	$=$	0.527
$i=9,$	\hat{u}_9	$=$	$16 - 16.264$	$=$	-0.264
$i=10,$	\hat{u}_{10}	$=$	$20 - 18.055$	$=$	1.945

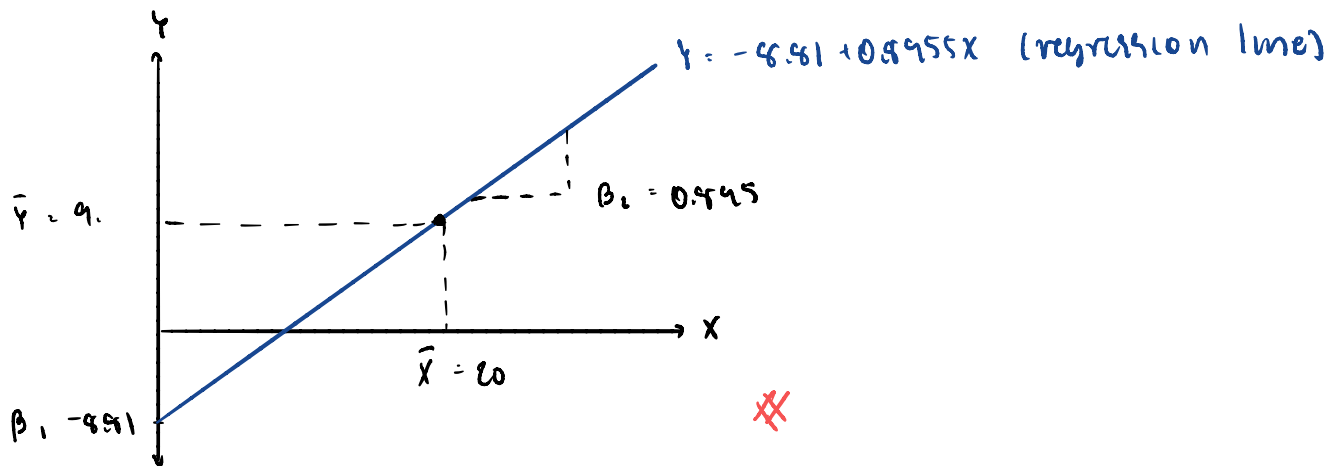
~~✗~~

$$\sum \hat{u}_i = -0.145 + 0.064 + 1.273 + 0.482 - 0.309 - 0.591 - 2.682 + 0.527 - 0.264 + 1.945$$

$$= 0.045 \approx 0$$

~~✗~~

(2.3)



Given $\bar{x} = 20, \bar{y} = 9.1$ → substitute \bar{x} into the regression function

$$y = -8.881 + 0.8955(20)$$

$$= 9.1 = \bar{y}$$

∴ the regression line passes (\bar{x}, \bar{y}) ~~X~~

(2.4)

from $\hat{y}_i = \beta_0 + \beta_1 x_i$

$$\hat{y}_i = -8.881 + 0.8955(14)$$

$$= 7.309 \quad \text{X}$$

(25)

$$\begin{aligned}
 \text{from } \text{var}(\hat{u}_i) &= \frac{\sum \hat{u}_i^2}{n-2} \\
 &= \frac{[(-0.145)^2 + 0.064^2 + 1.173^2 + 0.482^2 + (-0.309)^2 \\
 &\quad + (-0.591)^2 + (-2.682)^2 + 0.527^2 + (-0.146)^2 + 1.945^2]}{10-2} \\
 &= \frac{14.08173}{8} = 1.7602 \quad *
 \end{aligned}$$

$$\begin{aligned}
 \text{from } \text{var}(\hat{\beta}_1) &= \frac{\sum x_i^2 \sigma^2}{n \sum x_i^2} \\
 &= \frac{4440 (1.7602)}{10(440)} = 1.7762 \quad *
 \end{aligned}$$

$$\begin{aligned}
 \text{from } \text{var}(\hat{\beta}_2) &= \frac{\sigma^2}{\sum x_i^2} = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} \\
 &= \frac{1.7602}{[(10-20)^2 + (12-20)^2 + (14-20)^2 + (16-20)^2 + (18-20)^2 \\
 &\quad + (22-20)^2 + (24-20)^2 + (26-20)^2 + (28-20)^2 + (30-20)^2]} \\
 &= \frac{1.7602}{440} = 0.0040 \quad *
 \end{aligned}$$

(3.)

$$\begin{aligned} \text{from } \hat{\beta}_1 &= \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{n \sum x_i^2 - (\sum x_i)^2} \\ &= \bar{y} - \hat{\beta}_0 \bar{x} \end{aligned}$$

prove that this is an unbiased estimator

$$\begin{aligned} E(\hat{\beta}_1) &= E(\bar{y} - \hat{\beta}_0 \bar{x}) \\ &= E(\bar{y}) - \beta_0 E(\bar{x}) \\ &= \beta_1 + \beta_0 \bar{x} - \bar{x} \beta_0 \end{aligned}$$

so $E(\hat{\beta}_1) = \beta_1 \rightarrow$ unbiased

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$\hat{\beta}_1$ is normally distributed with:

$$\text{mean: } E(\hat{\beta}_1) = \beta_1$$