

EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1**Due date: 31 January 2020 before 11pm**

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

$$\begin{aligned} \text{a. } \sum_{i=1}^5 (a + bx_i) &= \sum_{i=1}^5 a + \sum_{i=1}^5 bx_i \\ &= 5a + (bx_1 + bx_2 + bx_3 + bx_4 + bx_5) \\ &= 5a + b(x_1 + x_2 + x_3 + x_4 + x_5) \end{aligned}$$

$$\begin{aligned} \text{b. } \sum_{y=0}^5 f(x+y) &= f(x) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5) \end{aligned}$$

$$\begin{aligned} \text{c. } \sum_{i=1}^{10} i^2 &= 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2 \\ &= 385 \end{aligned}$$

$$\begin{aligned} \text{d. } \sum_{x=1}^2 \sum_{y=2}^3 (2x+y) &= \sum_{x=1}^2 [(2x+2) + (2x+3)] = [(2(1)+2) + (2(2)+2)] + [(2(1)+3) + (2(2)+3)] \\ &= \sum_{x=1}^2 (2x+2) + \sum_{x=1}^2 (2x+3) = 22 \end{aligned}$$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

- a. Find the value of b

$$\begin{aligned} 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b &= 1 \\ b &= 0.125 \end{aligned}$$

- b. Find the answer for $P(X \leq 2)$

$$\begin{aligned} P(X \leq 2) &= 1 - P(X > 2) \\ &= 1 - [P(X=3) + P(X=4)] \end{aligned}$$

$$\begin{aligned} &= 1 - [(0.5(0.125)) + (0.25(0.125))] \\ &= 1 - [0.0625 + 0.03125] \\ &= 1 - 0.09375 \\ &= 0.90625 \end{aligned}$$

- c. Find the answer for $P(-2 \leq X \leq 3)$

$$\begin{aligned} P(-2 \leq X \leq 3) &= 1 - P(X=4) \\ &= 1 - (0.25 \times 0.125) \\ &= 1 - 0.03125 \\ &= 0.96875 \end{aligned}$$

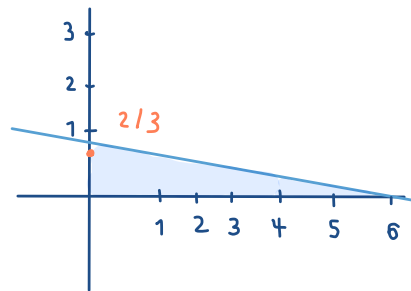
- d. Find the answer for $P(X \geq 1)$

$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - [P(X=0) + P(X=-1) + P(X=-2)] \\ &= 1 - [(2.25(0.125)) + 0.125 + (0.5(0.125))] \\ &= 1 - [0.28 + 0.125 + 0.06] \\ &= 1 - 0.47 \\ &= 0.53125 \end{aligned}$$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

- a. Plot graph for $f(x)$



- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} P(1 \leq x \leq 3) &= \int_1^3 f(x) dx = \left[-\frac{(3)^2}{18} + \frac{6(3)}{9} \right] - \left[-\frac{(1)^2}{18} + \frac{6(1)}{9} \right] \\ &= \left. -\frac{1}{18}x^2 + \frac{6}{9}x \right|_1^3 = \frac{16}{18} \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$\begin{aligned} P(x \geq 2) &= \int_2^3 f(x) dx = \left[-\frac{(3)^2}{18} + \frac{6(3)}{9} \right] - \left[-\frac{(2)^2}{18} + \frac{6(2)}{9} \right] \\ &= \left. -\frac{1}{18}x^2 + \frac{6}{9}x \right|_2^3 = \frac{7}{18} \end{aligned}$$

- d. Find the expected value of X

$$\begin{aligned} E(x) &= \int_{-\infty}^{\infty} x f(x) dx \\ &= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x dx \\ &= \left. -\frac{x^3}{27} + \frac{6x^2}{18} \right|_0^3 \\ &= -\frac{27}{27} + \frac{6 \cdot 9}{18} \\ &= -1 + 3 \\ &= 2 \end{aligned}$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

- a. Construct the joint probability distribution function (PDF) table of X and Y

a dice

a coin

$y \backslash x$	1	2	3	4	5	6	marginal (y)
1	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/2$
0	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/12$	$1/2$
marginal (x)	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	1

- b. Find the marginal probability distribution function (PDF) of X

The marginal prob of X

$$P(X=x) = \frac{1}{6}$$

- c. Find the marginal probability distribution function (PDF) of Y

The marginal prob of

$$P(Y=y) = \frac{1}{2}$$

- d. Find the conditional probability distribution function (PDF) of

X given Y is equal to 1 $P(X|Y=1)$

x	1	2	3	4	5	6
$P(X Y=1)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

- e. Find the expected value of X given Y is equal to 1 $\star 1/2 \rightarrow 0.5$

$$\begin{aligned}
 E(X|Y=1) &= \sum x_i P(X=x_i | Y=1) \\
 &= \frac{\sum x_i P(X=x_i, Y=1)}{P(Y=1)} \\
 &= \frac{1}{P(Y=1)} \sum x_i P(X=x_i, Y=1)
 \end{aligned}
 \left| \begin{aligned}
 &= \frac{1}{0.5} \left[(1 \cdot 1/12) + (2 \cdot 1/12) + (3 \cdot 1/12) \right. \\
 &\quad \left. + (4 \cdot 1/12) + (5 \cdot 1/12) + (6 \cdot 1/12) \right] \\
 &= \frac{7}{2}
 \end{aligned} \right.$$

- f. Find the variance of X given Y is equal to 1

$$\begin{aligned}
 \text{Var}(X|Y=1) &= \sum (X - E(X|Y=1))^2 \cdot P(X|Y=1) \\
 &= \left[\left((1 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((2 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((3 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) \right. \\
 &\quad \left. + \left((4 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((5 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) + \left((6 - \frac{7}{2})^2 \cdot \frac{1}{6} \right) \right] \\
 &= \left(\frac{25}{4} \cdot \frac{1}{6} \right) + \left(\frac{9}{4} \cdot \frac{1}{6} \right) + \left(\frac{1}{4} \cdot \frac{1}{6} \right) + \left(\frac{1}{4} \cdot \frac{1}{6} \right) + \left(\frac{9}{4} \cdot \frac{1}{6} \right) + \left(\frac{25}{4} \cdot \frac{1}{6} \right) \\
 &= \frac{10}{3}
 \end{aligned}$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{N} \sum_{i=1}^3 X_i\right) \\ &= \frac{1}{N} E(X_1 + X_2 + X_3) \\ &= \frac{1}{N} (E(X_1) + E(X_2) + E(X_3)) \\ &= \frac{1}{3} [\mu_X + \mu_X + \mu_X] \\ &= \frac{1}{3} \times 3\mu_X \\ &= \mu_X \end{aligned} \quad \left| \quad \begin{aligned} \text{var}(\bar{X}) &= \text{var}\left(\frac{1}{N} \sum_{i=1}^3 X_i\right) \\ &= \frac{1}{N^2} \text{var}(X_1 + X_2 + X_3) \\ &= \frac{1}{N^2} \left[\text{var}(X_1) + \text{var}(X_2) + \text{var}(X_3) \right. \\ &\quad \left. + 2\text{cov}(X_1, X_2) + 2\text{cov}(X_1, X_3) + 2\text{cov}(X_2, X_3) \right] \\ &= \frac{1}{3^2} \left[\frac{1}{4}\sigma_X^2 + \frac{1}{4}\sigma_X^2 + \frac{1}{4}\sigma_X^2 + \frac{1}{2}\sigma_X^2 + \frac{1}{2}\sigma_X^2 + \frac{1}{2}\sigma_X^2 \right] \\ &= \frac{1}{9} \cdot \frac{9}{4}\sigma_X^2 \\ &= \frac{\sigma_X^2}{4} \end{aligned}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{N} \sum_{i=1}^4 X_i\right) \\ &= \frac{1}{N} E(X_1 + X_2 + X_3 + X_4) \\ &= \frac{1}{N} [E(X_1) + E(X_2) + E(X_3) + E(X_4)] \\ &= \frac{1}{4} [\mu_X + \mu_X + \mu_X + \mu_X] \\ &= \mu_X \end{aligned} \quad \left| \quad \begin{aligned} \text{var}(\bar{X}) &= \text{var}\left(\frac{1}{N} \sum_{i=1}^4 X_i\right) \\ &= \frac{1}{N^2} \text{var}(X_1 + X_2 + X_3 + X_4) \\ &= \frac{1}{N^2} [\text{var}(X_1) + \text{var}(X_2) + \text{var}(X_3) + \text{var}(X_4)] \\ &= \frac{1}{4^2} [\sigma_X^2 + \sigma_X^2 + \sigma_X^2 + \sigma_X^2] \\ &= \frac{1}{16} \cdot 4\sigma_X^2 \\ &= 0.25\sigma_X^2 \end{aligned}$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that $\tilde{X}$ is an unbiased estimator of μ

$$\begin{aligned}\tilde{X} &= \frac{1}{4} (0.5X_1 + X_2 + 0.5X_3 + 2X_4) \\ E(\tilde{X}) &= E\left(\frac{1}{4} \sum_{i=1}^4 X_i\right) \\ &= \frac{1}{4} E(0.5X_1 + X_2 + 0.5X_3 + 2X_4) \\ &= \frac{1}{4} [E(0.5X_1) + E(X_2) + E(0.5X_3) + 2E(X_4)] \\ &= \frac{1}{4} (4\mu_X) \\ &= \mu_X \\ \tilde{X} \text{ is unbiased estimator for } \mu\end{aligned}$$

$$\begin{aligned}\text{Var}(\tilde{X}) &= \text{Var}\left(\frac{1}{4} \sum_{i=1}^4 X_i\right) \\ &= \frac{1}{4} \text{Var}(0.5X_1 + X_2 + 0.5X_3 + 2X_4) \\ &= \frac{1}{4} [\text{Var}(0.5X_1) + \text{Var}(X_2) \\ &\quad + \text{Var}(0.5X_3) + \text{Var}(2X_4)] \\ &= \frac{1}{4} [0.25\sigma^2 + \sigma^2 + 0.25\sigma^2 + 4\sigma^2] \\ &= \frac{5.5\sigma^2}{4} \\ &= 0.34\sigma^2\end{aligned}$$

- c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ ? Why?

$\bar{X}$ is more efficient estimator of μ_X than $\tilde{X}$ due to the smaller variance.