EE320 Exercise 2

Semester 2, 2014

Chapter 4: Matrix Algebra

1. Find $A^{-1}B$ when $A = \begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 7 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 2 & 0 \end{bmatrix}$

$$\left[-\frac{1}{17} \begin{bmatrix} -41 & 22 & -9 \\ 51 & -34 & 17 \\ 9 & -4 & -3 \end{bmatrix} \right]$$

2. A company sells 700 CDs , 400 DVDs, 200 CD players per week. The price per unit of these three products is \$4, \$6 and \$150 respectively. The costs per unit of these three products is \$3.25, \$4.75 and \$125 respectively. Find profit per week using matrix.
a) Total concept [\$6,025]
b) Per-unit concept [\$6,025]

3. Let

$$\begin{aligned} C &= C^* + bY \ ; \ 0<b<1 \\ I &= I^* - aR \ ; \ a>0 \\ M^s &= M^* \\ M^d &= K + dY - qR \ ; \ d, q>0 \end{aligned}$$

where

$$\begin{aligned} C &= \text{Consumption} \\ Y &= \text{Income} \\ I &= \text{Investment} \\ R &= \text{Interest rate} \\ M^s &= \text{Money supply} \\ M^d &= \text{Money demand} \end{aligned}$$

C^*, I^*, K and M^* are positive constant.

a) Write down the equation in term of matrix.
b) Solve for the equilibrium income (Y^*) and equilibrium interest rate (R^*), which make the two markets are in equilibrium, using Cramer's rule.
$[Y^* = \frac{(q)(C^*+I^*+a(M^*-K))}{(q)(1-b)+ad}, R^* = \frac{(d)(C^*+I^*)-(1-b)(M^*-K)}{(q)(1-b)+ad}]$
c) If money supply increase, what will happen to equilibrium interest rate (R^*)in b).

4. If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, B = $\begin{bmatrix} -2 & 3 \\ 4 & 0 \end{bmatrix}$, C = $\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$ Find $(ABC)^T$.

5. If A = $\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ Find $A^{-1}B$.

6. Let A = $\begin{bmatrix} 4 & 2 & 5 \\ 3 & 1 & 8 \\ 9 & 7 & 6 \end{bmatrix}$ Find determinant of A [det A =17]

7. If A = $\begin{bmatrix} 3 & -1 \\ -4 & 0 \\ 2 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 1 \\ -1 & -2 \\ 1 & 1 \end{bmatrix}$ Find $[A^T B]^{-1}$. $\left[[A^T B]^{-1} = \frac{1}{13} \begin{bmatrix} 0 & -13 \\ 1 & 12 \end{bmatrix} \right]$

8. Suppose there are only two countries, country A and B. (unit: million Baht)

$$\begin{aligned} \text{Country A} : C_A &= 150 + 0.4Y_A \\ M_A &= 0.6Y_A \\ I_A &= 250 \end{aligned}$$

$$\begin{aligned} \text{Country B} : C_B &= 100 + 0.5Y_B \\ M_B &= 0.4Y_B \\ I_B &= 200 \end{aligned}$$

$$\begin{aligned} \text{where} \quad C &= \text{Consumption} \\ Y &= \text{Income} \\ I &= \text{Investment} \\ M &= \text{Import} \end{aligned}$$

a) Determine equilibrium income of these two countries. [$Y_A^* = 571.43$ and $Y_B^* = 714.29$]
b) Determine balance of payments at equilibrium income from a) of these two countries. [A = 57.14 and B = 57.14]

9. Suppose there are three products in the market,

Demand for goods A : $q_A^d = 3 - P_A + P_B$ Supply for goods A : $q_A^s = P_A - 2$
Demand for goods B : $q_B^d = 3 - 2P_B + P_C$ Supply for goods B : $q_B^s = P_B - 1$
Demand for goods C : $q_C^d = 6 + 2P_A - P_C$ Supply for goods C : $q_C^s = 2P_C - 2$

Determine equilibrium prices and quantities for each product using Cramer's rule.
[$q_A = 3, q_B = 4, q_C = 10 \text{and} P_A = 5, P_B = 5, P_C = 6$]

10. From Y = C + I + G + (X - M)

$$\begin{aligned} \text{Let} \quad C &= a + bY, \quad a>0; 0<b<1 \\ M &= c + dY, \quad c>0; 0<d<1 \\ T &= fM, \quad f>0 \\ X &= X_0 \\ I &= I_0 \\ G &= T \end{aligned}$$

Find the equilibrium income (Y^*), equilibrium consumption (C^*) ,equilibrium import (M^*) using matrix.

[$Y^* = \frac{|A_1|}{|A|} = \frac{I_0+X_0+a+cf-c}{1-b-df+d}$,
$C^* = \frac{|A_2|}{|A|} = \frac{(-f+1)(ad-bc)+(a+bI_0+bX_0)}{1-b-df+d}$,
$M^* = \frac{|A_3|}{|A|} = \frac{(1-b)(c)+d(a+I_0+X_0)}{1-b-df+d}$]

11. Given the following information,

$$\begin{aligned} \text{Let} \quad C &= a + bY \\ I &= I_a + iY \\ G &= G_0 \\ T &= T_0 + tY \\ R &= R_0 \end{aligned}$$

a) Write down the equations in the form of matrix.
b) Solve for the solution using Cramer's rule.

12. Suppose you have to plan for the production of X_1 , X_2 and X_3, given the following information.
- To produce 1 ton of X_1, it requires 0.4 tons of X_2 and 0.2 tons of X_3
- To produce 1 ton of X_2, it requires 0.2 tons of X_1 and 0.1 tons of X_3
- To produce 1 ton of X_3, it requires 0.6 tons of X_1 and 0.4 tons of X_2

In addition, the consumers want to buy 10 million tons of X_1 ,20 million tons of X_2 and 10 million tons of X_3

a) Write the simultaneous equation in the form of matrix.
b) Prove by the adjoint matrix that inverse of coefficient matrix equals to
$$\frac{1}{36}\begin{bmatrix} 48 & 13 & 34 \\ 24 & 44 & 32 \\ 12 & 7 & 46 \end{bmatrix}$$
c) How many products should the company produce each?
[$X_1 = 30, X_2 = 40, X_3 = 20$]

13. In the beginning 2015, company A , B and C sell the homogeneous products and the market share of company A , B and C is 20% ,60% and 20% respectively.

At the end of 2015:

Company A can maintain their own consumers by 85% and lost the consumers to B and C by 5% and 10%, respectively.

Company B can maintain their own consumers by 55% and lost the consumers to A and C by 10% and 35%, respectively.

Company C can maintain their own consumers by 85% and lost the consumers to A and B by 10% and 5%, respectively.

a) Determine the market share of each company at the beginning of 2016

b) Suppose the performance of each company in 2016 is the same as in 2015, Determine the market share of each company in the beginning of 2017. Hint: Matrix S = market share , Matrix T = transition of market share, what is the meaning of TS?