Caution: Equilibrium notion requires the information of the *market* demand and *market* supply. Mostly, this information needs to be derived from the given individual demand and supply.

Definition:

1. Market demand is the total quantity demanded of each individual (consumer/buyer) combined.

$$Q^d = \sum_{i=1}^{N} Q_i^d$$

N - buyers

where Q^d is total quantity demanded at the market level.
Q_i^d is the quantity demanded for i-th individual.

2. Market supply is the total quantity supplied of each individual (firm/seller) combined.

$$Q^S = \sum_{i=1}^{M} Q_i^S$$

M - firms

where Q^S is total quantity demanded at the market level.
Q_i^S is the quantity demanded for i-th firm.

Two methods for the aggregation!

- Direction summation approach
  - Apply basic sigma operation
- Horizontal summation approach
  - Deriving market demand curve from individual demand curves
  - Find the mathematical equation that represent the derived market curve

See Example 3.E below

Example 3.E: Market demand

Consumer 1: $Q_1^d = 3 - P$

Consumer 2: $Q_2^d = 2 - P$

$$Q_1^d = \begin{cases} 0 & ; P \geq 3 \\ 3-P & ; P < 3 \end{cases}$$

$$Q_2^d = \begin{cases} 0 & ; P \geq 2 \\ 2-P & ; P < 2 \end{cases}$$

Direct Summation:

$$Q^d = \sum_{i=1}^{2} Q_i^d$$

$$= Q_1^d + Q_2^d$$

$$= 3-P + 2-P$$

$$= 5-2P$$

$P = 2.5 \rightarrow Q = 0$

$P = 2.5 \Rightarrow 3 - 2.5 = 0.5$

$P = 2.5 \Rightarrow 2 - 2.5 = -0.5$

$\} = 0$

$\underline{P = 2.5} \rightarrow \#2$

$P > 2 \rightarrow \#2$ is not in the market.

heterogenous demand.

$$Q^d = Q_1^d + Q_2^d = \begin{cases} \overset{\#1}{0} + \overset{\#2}{0} \Rightarrow 0 & ; \; P \geq 3 \\ \overset{\#1}{\underbrace{3-P}_{>0}} + \underbrace{0}_{\text{D fn \#2 consumer}} & ; \; 2 \leq P < 3 \\ \overset{\#1}{\underbrace{3-P}_{>0}} + \overset{\#2}{\underbrace{2-P}_{>0}} = \underline{5-2P} & ; \; P < 2 \end{cases}$$

$Q_i^d = 3-P$ homogenous demand

10 individual $\rightarrow$ has the same demand f^2

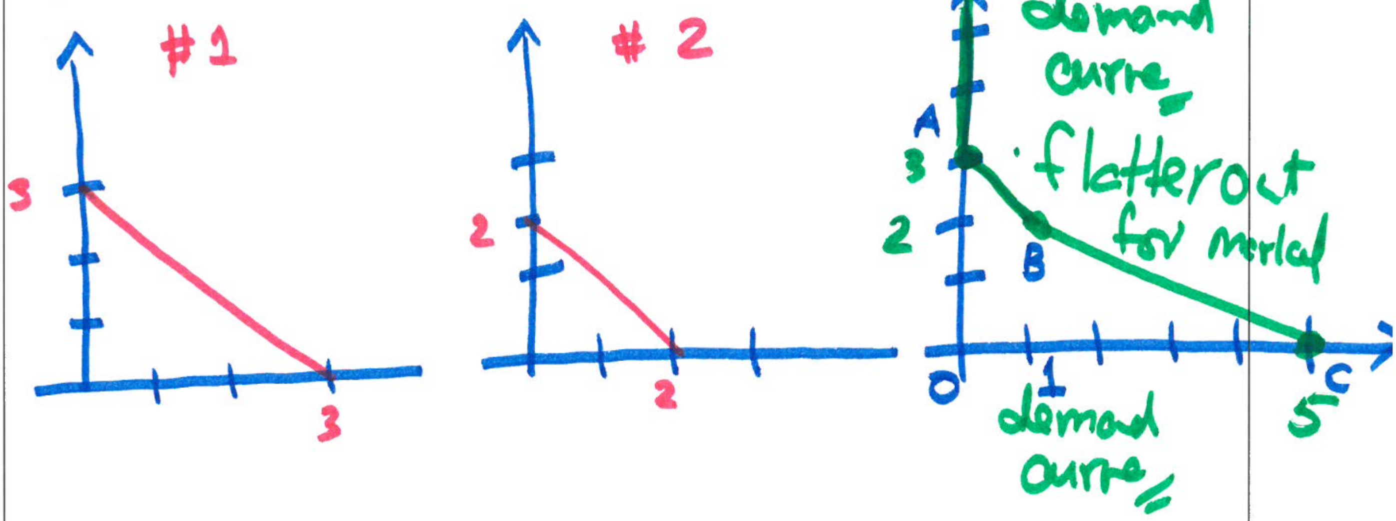
$$\Rightarrow \sum_{i=1}^{N} Q_i^d$$

$$\Rightarrow Q^d = N(Q_i^d) = 10(3-P) = 30-10P \; \#$$

$Q_1^d = 3 - P \; ; \; Q_2^d = 2 - P$

Deriving market demand curve from individual demand curves.

$P = 3 - Q_1^d \; ; \; P = 2 - Q_2^d$



What is the mathematical equation that represents the market demand curve when using the horizontal summation method? Derive the equation/function.

$\overrightarrow{AB} \Rightarrow -1 \Rightarrow Q \in [0,1] \; ; \; P = 3 - Q$

$\overrightarrow{BC} \Rightarrow -\frac{1}{2} \qquad Q > 1 \; ; \; P = \frac{5}{2} - \frac{1}{2}Q$

$$P = \begin{cases} 3 - Q & ; 0 \leq Q \leq 1 \\ \frac{5}{2} - \frac{1}{2}Q & ; \cancel{Q > 1} \quad 1 < Q \leq 5 \end{cases}$$

Example 3.F: (market supply):

A market has two firms, A and B. Each has the supply equation given by,

$Q_a = P - 15$

Firm A: P = 15 + Qa $\Rightarrow Q_a = 0 \; ; \; P \leq 15$

Firm B: Qb = 0.5P - 5 $\Rightarrow Q_b = 0 \; ; \; P \leq 10$

Derive the market supply equation

$$Q^S = \sum Q_i^S = Q_a + Q_b =$$

$$Q^S = \begin{cases} \underset{\#B}{0} + \underset{\#A}{0} \; ; & 0 < P \leq 10 \\ \underset{\#B}{0.5P-5} + \underset{\#A}{0} \; ; & 10 < P \leq 15 \\ \underset{\#B}{0.5P-5} + \underset{\#A}{P-15} \; ; & P > 15 \end{cases}$$

$= 1.5P - 20$

$$Q^S = \begin{cases} 0 & ; 0 < P \leq 10 \; ; \text{ no firm} \\ 0.5P-5 & ; 10 < P \leq 15 \; ; \text{ one firm } \#B \\ 1.5P-20 & ; P > 15 \; ; \text{ two firm } \#A, \#B \end{cases}$$

$\rightarrow p^* \Rightarrow Q^d = Q^S$

Example 3.G: Solve for the market equilibrium using the information in **Example 3.E** and **Example 3.F**. Justify your answer!

$$Q^d = \begin{cases} 0 & ; \quad p \geq 3 \\ 3-p & ; \quad 2 \leq p < 3 \\ 5-2p & ; \quad p < 2 \end{cases}$$

$\rightarrow$ highest maximum price for consumer.

$$Q^S = \begin{cases} 0 & ; \quad 0 \leq p \leq 10 \\ 0.5p-5 & ; \quad 10 \leq p \leq 15 \\ 1.5p-20 & ; \quad p > 15 \end{cases}$$

$\rightarrow$ lowest minimum price for producer

max Price $<$ min Price

No way.

$\not\Rightarrow Q^d = Q^S$

Impossible to have p^* such that $Q^d = Q^S$ in the positive quadrant

Some extensions: Government interventions

Taxation

- The first type of government intervention is *taxation*
- Micro-market model allows us to understand the impact of commodity tax.
- Basically, there are two types of commodity tax.
  - Unit tax ➔ \$t per each unit of production.
  - Ad-valorem tax ➔ t% of value.
- Let's start from the impact of unit tax. Then, we will come back to the analysis on Ad-valorem tax later.
- The purpose of this part is to show:
  - A rigorous way to analyze the impact taxation on price and allocation in the market equilibrium.
  - Showing that the implication of tax on the welfare, and distribution of welfare cost among stakeholders in the market.

Impact of the unit taxation

The basic set up of linear market model typically takes the form:

Demand: $p \quad = a - bQ^d \quad ; \; a \geq 0, \quad b \geq 0.$

Supply $: p \quad = c + dQ^s \quad ; \; d \geq 0.$

- Equilibrium can be then solved by imposing the market clearing condition, i.e. $Q^d = Q^s$

When the government imposes tax, the followings happen.

1. With the imposition of tax, *price that consumer's paying will no longer be equal to price that producer's receiving*.
   - The single "p" concept that underlies the basic set up of linear market model would no longer be sensible!
   - So, to analyze the impact of taxation, we need to make it clear from now that
     - $p^s = price\ that\ producer\ gets$
     - $p^d = price\ that\ consumer\ pays$
     - $p^s \neq p^d$ ➔ price gap *or* price wedges

tax imposed on consumer

$p^d = p^s + t$

Behavior of buyers and suppliers would focus at their perceived prices.

That is, the market model needs to be rewritten in the following way.

tax on produc

$p^s = p^d - t$

Demand: $p^d = a - bQ^d \quad ; \; a \geq 0, \quad b \geq 0.$

Supply $: p^s = c + dQ^s \quad ; \; d \geq 0.$

2. With the imposition of tax, equilibrium allocation would be affected!

   - VERY SIMPLE, we solve for equilibrium by imposing that $Q^d = Q^s$.
   - BUT!!!!

Example 3.H Can we solve for the equilibrium by simply imposing the only market clearing condition?

Solvable? If not, what else do we need?

Example 3.I Suppose that demand/supply equation can be given by

$$P^d = 80 - 3Q^d \rightarrow \text{steeper}$$

$$P^s = 10 + 2Q^s \rightarrow \text{flatter}$$

- Find the market equilibrium.

$p^d = p^s \Rightarrow \quad 80 - 3Q = 10 + 2Q$

$70 = 5 \cdot Q$

$Q^* = 14$ units

$p^* = 10 + 2(14) = 10 + 28 = 38\$$

- Suppose that government imposes tax on producer equal to \$5 per unit, determine market equilibrium under taxation.

→ "p" ⇒ clear the market: $Q^d = Q^s$

$3/5 = 60\%$ $\boxed{p^s = p^d - 5}$

5 → 3 consumer; → 2 producer

$p^d = 80 - 3Q^d$

$p^s = p^d - 5 = 10 + 2Q^s$

$p^d = 15 + 2Q^s$

$80 - 3Q = 15 + 2Q$

$65 = 5 \cdot Q$

$Q^* = 13$ units

$p^{d*} = 41\$$

$p^{s*} = 41 - 5 = 36$

$\frac{2}{5} = 40\%$

before $p^d = 10 \Rightarrow$ good ; $p^d < 10 \rightarrow$ out

p^d, p^s

$p^s = 5$; need at least \$ K fan

$TR = 13 \times 5 = 65\$$

consumer price