Section 2.5 Proof of statements in the form of "$p \vee q$"

Since $(p \vee q) \leftrightarrow (\sim p \to q)$, by Law of Implication, we can then prove $\sim p \to q$ instead of $p \vee q$. To prove $\sim p \to q$, we can assume $\sim p$ and show that q is true. Moreover, since $(p \vee q) \equiv (q \vee p)$ and $(q \vee p) \leftrightarrow (\sim q \to p)$, hence there are two ways to prove $p \vee q$.

1) Assume $\sim p$ and show that q is true.
2) Assume $\sim q$ and show that p is true

Example 1: Prove that if $(\sim x \wedge y) \to \sim z$, $\sim (x \vee y) \to w$, and $\sim x$, then $\sim z \vee w$.

Example 2: Prove that if $a \vee (b \wedge \sim c)$, $\sim a$, $b \to (d \to e)$, and $\sim c \to (x \to y)$, then $[a \vee x \vee d] \to (y \vee e)$.

Example 3: Prove that if $(x \wedge y \wedge \sim z) \to \sim y$, then $\sim x \vee (y \to z)$.

Section 2.6 Proof of statements in the form of " $p \leftrightarrow q$ "

Since $(p \leftrightarrow q) \leftrightarrow [(p \to q) \wedge (q \to p)]$, by Law of Equivalence, we can then prove $(p \to q) \wedge (q \to p)$ instead of $p \leftrightarrow q$. To prove $(p \to q) \wedge (q \to p)$, we have to prove that $p \to q$ is true and also $q \to p$ is true by the method of proving the statements in the form of $p \to q$.

Example 1: Prove that if $a \to (b \wedge c)$, $(d \wedge c) \to (e \wedge \sim a)$, $a \vee (d \to \sim e)$, and $d \wedge (b \to e)$, then $\sim e \leftrightarrow \sim a$.

Example 2: Prove that $[p \to (q \to r)] \leftrightarrow [\sim (p \to r) \to (q \to \sim p)]$.

Section 2.7 Proof of statements in the form of "$(p \vee q) \to r$"

Note that

$$\begin{aligned}
[(p \vee q) \to r] &\equiv [\sim (p \vee q) \vee r] && \text{Law of Implication} \\
&\equiv [(\sim p \wedge \sim q) \vee r] && \text{De Morgan's Laws} \\
&\equiv [(\sim p \vee r) \wedge (\sim q \vee r)] && \text{Distributive Laws} \\
&\equiv [(p \to r) \wedge (q \to r)] && \text{Law of Implication}
\end{aligned}$$

Hence, we can prove $(p \to r) \wedge (q \to r)$ instead of $(p \vee q) \to r$. In order to show that $(p \to r) \wedge (q \to r)$ is true, we must show that $p \to r$ is true and also $q \to r$ is true.

Example 1: Let m and n be integers. Prove that *if* m and n are even *or* m and n are odd, *then* the sum of m and n is even.

We have now seen many examples of integers that can be expressed as $2x$ for some integer x. There are precisely the even integers, of course. However, some integers can also be written as $3x$ or $4x$ or as $-5x$ for some integer x. In general, for integers a and b with $a \neq 0$, we say that a **divides** b if there is an integer c such that $b = ac$. In this case, we write $a \mid b$.

If $a \mid b$, then we also say b is a **multiple** of a and that a is a **divisor** of b. Thus every even integer is a multiple of 2. If a does not divide b, then we write $a \mid b$.

Example 2: Let m be any integer. Prove that if 4 divides m or 6 divides m, then $m^2 - 1$ is odd.