Solution to Homework 3

CHAPTER 8

5. a. To optimize this portfolio one would need:

n = 60 estimates of means

n = 60 estimates of variances

$\frac{n^2-n}{2} = 1{,}770$estimates of covariances

Therefore, in total: $\frac{n^2+3n}{2} = 1{,}890$estimates

b. In a single index model: $r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + e_i$

Equivalently, using excess returns: $R_i = \alpha_i + \beta_i R_M + e_i$

The variance of the rate of return can be decomposed into the components:

(l) The variance due to the common market factor: $\beta_i^2 \sigma_M^2$

(2) The variance due to firm specific unanticipated events: $\sigma^2(e_i)$

In this model: $Cov(r_i, r_j) = \beta_i \beta_j \sigma$

The number of parameter estimates is:

n = 60 estimates of the mean $E(r_i)$

n = 60 estimates of the sensitivity coefficient β_i

n = 60 estimates of the firm-specific variance $\sigma^2(e_i)$

1 estimate of the market mean $E(r_M)$

1 estimate of the market varianceσ_M^2

Therefore, in total, 182 estimates.

The single index model reduces the total number of required estimates from 1,890 to 182. In general, the number of parameter estimates is reduced from:

$$\left(\frac{n^2 + 3n}{2}\right) \text{ to } (3n + 2)$$

7. a. The two figures depict the stocks' security characteristic lines (SCL). Stock A has higher firm-specific risk because the deviations of the observations from the SCL are larger for Stock A than for Stock B. Deviations are measured by the vertical distance of each observation from the SCL.

b. Beta is the slope of the SCL, which is the measure of systematic risk. The SCL for Stock B is steeper; hence Stock B's systematic risk is greater.

c. The R^2 (or squared correlation coefficient) of the SCL is the ratio of the explained variance of the stock's return to total variance, and the total variance is the sum of the explained variance plus the unexplained variance (the stock's residual variance):

$$R^2 = \frac{\beta_i^2 \sigma_M^2}{\beta_i^2 \sigma_M^2 + \sigma^2(e_i)}$$

Since the explained variance for Stock B is greater than for Stock A (the explained variance is $\beta_B^2 \sigma_M^2$, which is greater since its beta is higher), *and* its residual variance $\sigma^2(e_B)$ is smaller, its R^2 is higher than Stock A's.

d. Alpha is the intercept of the SCL with the expected return axis. Stock A has a small positive alpha whereas Stock B has a negative alpha; hence, Stock A's alpha is larger.

e. The correlation coefficient is simply the square root of R^2, so Stock B's correlation with the market is higher.

CHAPTER 9

8. The appropriate discount rate for the project is:

$r_f + \beta \times [E(r_M) - r_f] = .08 + [1.8 \times (.16 - .08)] = .224 = 22.4\%$

Using this discount rate:

$NPV = -\$40 + \int_{t=1}^{10} \frac{\$15}{1.224^t} = -\$40 + [\$15 \times \text{Annuity factor (22.4\%, 10 years)}] = \$18.09$

The internal rate of return (IRR) for the project is 35.73%. Recall from your introductory finance class that NPV is positive if IRR > discount rate (or, equivalently, hurdle rate). The highest value that beta can take before the hurdle rate exceeds the IRR is determined by:

$.3573 = .08 + \beta \times (.16 - .08) \Rightarrow \beta = .2773/.08 = 3.47$

20. $r_1 = 19\%$; $r_2 = 16\%$; $\beta_1 = 1.5$; $\beta_2 = 1$

a. To determine which investor was a better selector of individual stocks we look at abnormal return, which is the ex-post alpha; that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (risk-free rate and market rate of return) we cannot determine which investor was more accurate.

b. If $r_f = 6\%$ and $r_M = 14\%$, then (using the notation alpha for the abnormal return):

$\alpha_1 = .19 - [.06 + 1.5 \times (.14 - .06)] = .19 - .18 = 1\%$

$\alpha_2 = .16 - [.06 + 1 \times (.14 - .06)] = .16 - .14 = 2\%$

Here, the second investor has the larger abnormal return and thus appears to be the superior stock selector. By making better predictions, the second investor appears to have tilted his portfolio toward underpriced stocks.

c. If $r_f = 3\%$ and $r_M = 15\%$, then:

$\alpha_1 = .19 - [.03 + 1.5 \times (.15 - .03)] = .19 - .21 = -2\%$

$\alpha_2 = .16 - [.03 + 1 \times (.15 - .03)] = .16 - .15 = 1\%$

Here, not only does the second investor appear to be the superior stock selector, but the first investor's predictions appear valueless (or worse).

CHAPTER 15

7.

Maturity	Price	YTM	Forward Rate
1	$943.40	6.00%	
2	$898.47	5.50%	$(1.055^2/1.06) - 1 = 5.0\%$
3	$847.62	5.67%	$(1.0567^3/1.055^2) - 1 = 6.0\%$
4	$792.16	6.00%	$(1.06^4/1.0567^3) - 1 = 7.0\%$

8. The expected price path of the 4-year zero coupon bond is shown below. (Note that we discount the face value by the appropriate sequence of forward rates implied by this year's yield curve.)

Beginning of Year	Expected Price	Expected Rate of Return
1	$792.16	($839.69/$792.16) – 1 = 6.00%
2	$\frac{\$1{,}000}{1.05 \times 1.06 \times 1.07} = \$839.69$	($881.68/$839.69) – 1 = 5.00%
3	$\frac{\$1{,}000}{1.06 \times 1.07} = \$881.68$	($934.58/$881.68) – 1 = 6.00%
4	$\frac{\$1{,}000}{1.07} = \$934.58$	($1,000.00/$934.58) – 1 = 7.00%

9. If expectations theory holds, then the forward rate equals the short rate, and the one year interest rate three years from now would be

$$\frac{(1.07)^4}{(1.065)^3} - 1 = .0851 = 8.51\%$$

11. a. $P = \frac{\$9}{1.07} + \frac{\$109}{1.08^2} = \$101.86$

b. The yield to maturity is the solution for y in the following equation:

$$\frac{\$9}{1+y} + \frac{\$109}{(1+y)^2} = \$101.86$$

[Using a financial calculator, enter n = 2; FV = 100; PMT = 9; PV = –101.86; Compute i] YTM = 7.958%

c. The forward rate for next year, derived from the zero-coupon yield curve, is the solution for f_2 in the following equation:

$$1 + f_2 = \frac{(1.08)^2}{1.07} = 1.0901 \Rightarrow f_2 = 0.0901 = 9.01\%.$$

Therefore, using an expected rate for next year of $r_2 = 9.01\%$, we find that the forecast bond price is:

$$P = \frac{\$109}{1.0901} = \$99.99$$

d. If the liquidity premium is 1% then the forecast interest rate is:

$$E(r_2) = f_2 - \text{liquidity premium} = 9.01\% - 1.00\% = 8.01\%$$

The forecast of the bond price is:

$$\frac{\$109}{1.0801} = \$100.92$$