EE325: Homework 1 Solution

Chapter 1 Problem 1

(i) Ideally, we could randomly assign students to classes of different sizes. That is, each student is assigned a different class size without regard to any student characteristics such as ability and family background. For reasons we will see in Chapter 2, we would like substantial variation in class sizes (subject, of course, to ethical considerations and resource constraints).

(ii) A negative correlation means that a larger class size is associated with lower performance. We might find a negative correlation because a larger class size actually hurts performance. However, with observational data, there are other reasons we might find a negative relationship. For example, children from more affluent families might be more likely to attend schools with smaller class sizes, and affluent children generally might score better on standardized tests. Another possibility is that, within a school, a principal might assign the better students to smaller classes. Or, some parents might insist their children to be placed in smaller classes, and these same parents tend to be more involved in their children's education.

(iii) Given the potential for confounding factors – some of which are listed in (ii) – finding a negative correlation would not be strong evidence that smaller class sizes actually lead to better performance. Some way of controlling for the confounding factors is needed, and this is the subject of multiple regression analysis.

Appendix C Problem 1

C.1 (i) This is just a special case of what we covered in the text, with $n = 4$: $E(\bar{Y}) = \mu$ and $\mathrm{Var}(\bar{Y}) = \sigma^2/4$.

(ii) $E(W) = E(Y_1)/8 + E(Y_2)/8 + E(Y_3)/4 + E(Y_4)/2 = \mu[(1/8) + (1/8) + (1/4) + (1/2)] = \mu(1 + 1 + 2 + 4)/8 = \mu$, which shows that W is unbiased. Because the Y_i are independent,

$$\begin{aligned}\mathrm{Var}(W) &= \mathrm{Var}(Y_1)/64 + \mathrm{Var}(Y_2)/64 + \mathrm{Var}(Y_3)/16 + \mathrm{Var}(Y_4)/4 \\ &= \sigma^2[(1/64) + (1/64) + (4/64) + (16/64)] = \sigma^2(22/64) = \sigma^2(11/32).\end{aligned}$$

(iii) Because $11/32 > 8/32 = 1/4$, $\mathrm{Var}(W) > \mathrm{Var}(\bar{Y})$ for any $\sigma^2 > 0$, so $\bar{Y}$ is preferred to W when each is unbiased.

Appendix C Problem 3

C.3 (i) $E(W_1) = [(n-1)/n]E(\bar{Y}) = [(n-1)/n]\mu$, and so $\text{Bias}(W_1) = [(n-1)/n]\mu - \mu = -\mu/n$. Similarly, $E(W_2) = E(\bar{Y})/2 = \mu/2$, and so $\text{Bias}(W_2) = \mu/2 - \mu = -\mu/2$. The bias in W_1 tends to zero as $n \to \infty$, while the bias in W_2 is $-\mu/2$ for all n. This is an important difference.

(ii) $\text{plim}(W_1) = \text{plim}[(n-1)/n] \cdot \text{plim}(\bar{Y}) = 1 \cdot \mu = \mu$ and $\text{plim}(W_2) = \text{plim}(\bar{Y})/2 = \mu/2$. Because $\text{plim}(W_1) = \mu$ and $\text{plim}(W_2) = \mu/2$, W_1 is consistent whereas W_2 is inconsistent.

(iii) $\text{Var}(W_1) = [(n-1)/n]^2\text{Var}(\bar{Y}) = [(n-1)^2/n^3]\sigma^2$ and $\text{Var}(W_2) = \text{Var}(\bar{Y})/4 = \sigma^2/(4n)$.

(iv) Because $\bar{Y}$ is unbiased, its mean squared error is simply its variance. On the other hand, $\text{MSE}(W_1) = \text{Var}(W_1) + [\text{Bias}(W_1)]^2 = [(n-1)^2/n^3]\sigma^2 + \mu^2/n^2$. When $\mu = 0$, $\text{MSE}(W_1) = \text{Var}(W_1) = [(n-1)^2/n^3]\sigma^2 < \sigma^2/n = \text{Var}(\bar{Y})$ because $(n-1)/n < 1$. Therefore, $\text{MSE}(W_1)$ is smaller than $\text{Var}(\bar{Y})$ for μ close to zero. For large n, the difference between the two estimators is trivial.