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EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

$\bar{X} = \frac{63 + 72 + \dots + 90}{8} = 77.625$ $\bar{Y} = \frac{2.8 + 3.4 + \dots + 3.7}{8} = 3.2125$

3.2125

Table 1.a 77.625

Student	Y_i	X_i	$Y_i - \bar{Y}$	$X_i - \bar{X}$	$(X_i - \bar{X})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$	u_i
1	2.8	63	-0.4125	-14.625	213.890625	6.0328125	0.0857
2	3.4	72	0.1875	-5.625	31.640625	-1.0546875	0.3791
3	3	78	-0.2125	0.375	0.140625	-0.0796875	-0.2253
4	3.5	81	0.2875	3.375	11.390625	0.9703125	0.1725
5	3.6	87	0.3875	9.375	87.890625	3.6328125	0.0681
6	3.0	75	-0.2125	-2.625	6.890625	0.5578125	-0.1231
7	2.7	75	-0.5125	-2.625	6.890625	1.3453125	-0.4231
8	3.7	90	0.4875	12.375	153.140625	6.0328125	0.0659
					511.875	17.4375	0.0002

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1. (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{Cov(X,Y)}{Var(X)} = \frac{\sum_i (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_i (X_i - \bar{X})^2} = \frac{17.4375}{511.875} = 0.034065934065934 \approx 0.0341$$

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} = 3.2125 - 0.034(77.625) = 0.5681318681318 68 \approx 0.5681$$

1.2 For each observation i, find $\widehat{Y_i}$ and $\widehat{u_i}$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 X_i = 0.5681 + 0.0341 X_i$$

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$\hat{u}_i = Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i$$

$$\hat{u}_i = Y_i - 0.5681 - 0.0341 X_i$$

$$\sum_{i=1}^{8} \hat{u}_i = (2.8 - 0.5681 - 0.0341(63)) + (3.4 - 0.5681 - 0.0341(72)) + \dots = 0.0002 \approx 0$$

$SSR = \sum(\hat{y} - \bar{y})^2 = 1.02875$

$SST = \sum(x_i - \bar{x})^2 = 551.875$

1.3 Find $var(\widehat{u_i}), var(\widehat{\beta_0}), var(\widehat{\beta_1})$

$$Var(\hat{u}_i) = \frac{SSR}{n-2} = \frac{\sum(\hat{y}-\bar{y})^2}{8-2} = \frac{1.02875}{6} = 0.1715 = \sigma^2$$

$$Var(\hat{\beta}_0) = \frac{\sigma^2 n^{-1} \sum_{i=1}^{n} x_i^2}{SST} = \frac{(0.1715)(48717)}{8(551.875)} = 2.0398$$

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{SST} = \frac{0.1715}{551.875} = 0.00031076$$

$\bar{X} = 20$

$\bar{Y} = 9.1$

2. Data is listed in the table

X_i	Y_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$	u_i
10	0	-10	-9.1	-0.1455
12	2	-8	-7.1	0.0636
14	5	-6	-4.1	1.2727
16	6	-4	-3.1	0.4818
18	7	-2	-2.1	-0.3091
22	10	2	0.9	-0.8909
24	10	4	0.9	-2.6818
26	15	6	5.9	0.5273
28	16	8	6.9	-0.2636
30	20	10	10.9	1.9455

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i, u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\bar{X} = \frac{10 + 12 + \dots + 30}{10} = 20 \qquad \bar{Y} = \frac{0 + 2 + \dots + 20}{10} = 9.1$$

$$\beta_1 = \frac{Cov(X,Y)}{Var(X)} = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2} = \frac{-10(-9.1) + (-8)(-7.1) + \dots + 10(10.9)}{(-10)^2 + (-8)^2 + \dots + 10^2} = \frac{394}{440} = 0.8955$$

$$\beta_0 = \bar{Y} - \beta_1 \bar{X} = 9.1 - (0.8955)20 = -8.81$$

2.2 Find the value of $\widehat{Y_i}$ and $\widehat{u_i}$. Show that $\sum_{i=0}^{N} \hat{u}_i = 0$.

$$\left.\begin{aligned} \hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_i \\ \hat{u}_i &= y_i - \hat{y}_i \end{aligned}\right\} \quad \hat{u}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$$

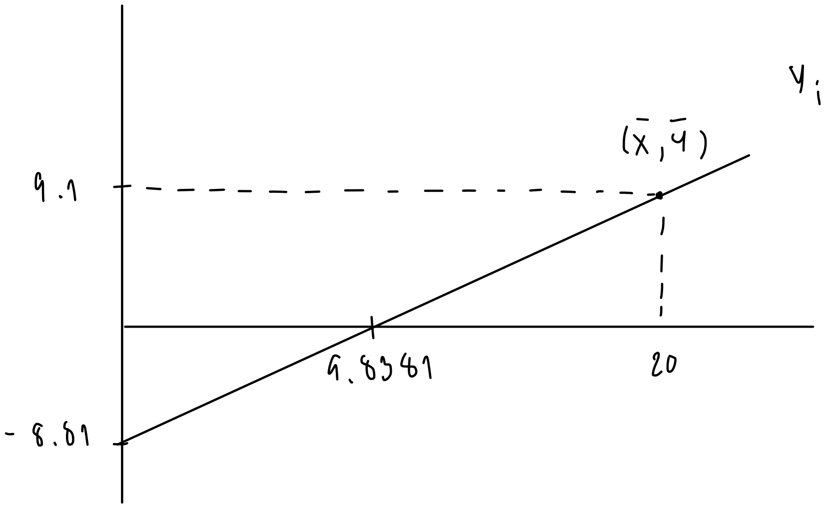
$$= y_i + 8.81 - 0.8955 x_i$$

$$\Sigma \hat{u}_i = 10 + 8.81 - 0.8955(10)) + (2 + 8.81 - 0.8955(12)) + \ldots + (20 + 8.81 - 0.8955(30))$$

$$= 0$$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

$$(\bar{x}, \bar{y}) = (20, 9.1)$$



$$y_i = -8.81 + 0.8955 x_i$$

$$y_i = -8.81 + 0.8955(20)$$

$$y_i = -8.81 + 17.91 = 9.1$$

$$y_i = 9.1 = \bar{y}$$

2.4 If X_i =16, what is the predicted Y?

$$E(Y) = -8.81 + 0.8955(16)$$

$$= 5.518$$

2.5 Find $var(\widehat{u_i})$, $var(\widehat{\beta_0})$, $var(\widehat{\beta_1})$

$$SSR = \Sigma(y_i - \bar{y})^2 = 366.9$$

$$SST = \Sigma(x_i - \bar{x})^2 = 440$$

$$Var(\hat{u}_i) = \sigma^2 = \frac{SSR}{n-2} = \frac{366.9}{8} = 45.8625$$

$$Var(\hat{\beta}_0) = \frac{\sigma^2 n^{-1} \sum_{i=1}^{n} x_i^2}{SST} = \frac{45.8625(4440)}{440(10)} = 46.2794$$

$$Var(\hat{\beta}_1) = \frac{\sigma^2}{SST} = \frac{45.8625}{440} = 0.104233$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i \,,$$

Where $u_i \sim NIID(0, \sigma^2)$**.** Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$y_i = \beta_1 x_i + u_i$$

$$u_i = y_i - \beta_1 x_i$$

minimize error : $\arg\min \sum (y_i - \hat{\beta}_1 x_i)^2$

F.O.C wrt $\hat{\beta}_1$; $\dfrac{\partial \sum (y_i - \hat{\beta}_1 x_i)^2}{\partial \hat{\beta}_1} = 0$

$$\sum 2(y_i - \hat{\beta}_1 x_i)(-x_i) = 0$$

$$\sum (y_i x_i - \hat{\beta}_1 x_i^2) = \frac{0}{-2} = 0$$

$$\sum y_i x_i - \hat{\beta}_1 \sum x_i^2 = 0$$

$$\hat{\beta}_1 = \frac{\sum y_i x_i}{\sum x_i^2}$$

substitute $y_i = \beta_1 x_i + u_i$; $\hat{\beta}_1 = \dfrac{\sum x_i (\beta_1 x_i + u_i)}{\sum x_i^2}$

$$\hat{\beta}_1 = \frac{\sum \beta_1 \cancel{x_i^2}}{\sum \cancel{x_i^2}} + \frac{\sum x_i u_i}{\sum x_i^2}$$

by SLR 4 ; $E(u_i | x_i) = 0$

$$E(\hat{\beta}_1) = E(\beta_1) + E\left(\frac{\sum x_i u_i}{\sum x_i^2}\right)$$

$$E(\hat{\beta}_1) = \beta_1$$