Quiz 2

1. Let $A = \{0, 1, 2\}$ be the domain of variable a. Let the set of real number $\mathbb{R}$ be the domain of variable y. Consider the predicate

$$P(y): \quad \forall a \in A, \; \Big[\; (a < y) \vee (a + y > 10) \; \Big].$$

(a) Write the negation of $P(y)$ (without using negation symbol "$\sim$" in the final answer).

(b) Determine the **truth set** of the predicate $P(y)$. Explain your answer.

Solution:

(a) Write the negation of $P(y)$ (without using negation symbol "$\sim$" in the final answer).

$$\begin{aligned} \sim P(y) \quad &\equiv \quad \exists a \in A, \; \sim [(a < y) \vee (a + y > 10)]\,. \\ &\equiv \quad \exists a \in A, \; [\sim (a < y) \wedge \sim (a + y > 10)]\,. \\ &\equiv \quad \exists a \in A, \; (a \geq y) \wedge (a + y \leq 10). \end{aligned}$$

(b) Determine the **truth set** of the above statement. Explain your answer.

Consider each value of $a \in \{0, 1, 2\}$.

(i) For $a = 0$, we have $(0 < y) \vee (y > 10)$,
which is true for $y > 0$ or $y > 10$. That is, $\boxed{y \in (0, \infty)}$.

(ii) For $a = 1$, we have $(1 < y) \vee (1 + y > 10)$,
which is true for $y > 1$ or $y > 9$. That is, $\boxed{y \in (1, \infty)}$.

(iii) For $a = 2$, we have $(2 < y) \vee (2 + y > 10)$,
which is true for $y > 2$ or $y > 8$. That is, $\boxed{y \in (2, \infty)}$.

The given statement has the universal quantifier $\forall$, so we need this to be true for all values of $a \in A = \{0, 1, 2\}$ and the value of y has to be an element in the intersection of all cases from (i)-(iii), i.e.

$$y \in (0, \infty) \cap (1, \infty) \cap (2, \infty) = (2, \infty).$$

That is, the truth set for $P(y)$ is the interval $\boxed{(2, \infty)}$.