EE325 Introductory Econometrics (Section 1 semester 1/2020)

Assignment 1

Instruction: Write your answer in either paper or digital paper. However, if you write on paper, please scan it and save as a PDF file. Submission is via BE-Moodle as a PDF file for both cases. (Please keep the file below 10MB as that is the maximum per file capacity for student.)

Due date: Tuesday, September 1,2020 (Before class starts at 11.00 A.M.)

1. Explain the difference between cross-sectional, time-series, and panel data.

The key difference is that, while panel data include multiple aspects of observation for different time, cross-sectional data are collected in a single period of time and time-series do the opposite, having limited variables from different times.

2. Let X and Y be discrete random variables with the joint PDF displayed in the following table. Answer the following questions.

		X		
		1	2	3
Y	2	0.1	0.2	0.1
	4	0.3	0.2	**a**

2.1 Find the value of **a** in the table and explain why.

0.1 since total probability must = 1 #

2.2 Find $E(X)$ and $E(Y)$.

$E(X) = 1(0.4) + 2(0.4) + 3(0.2)$

$= 0.4 + 0.8 + 0.6$

$= 1.8$ #

$E(Y) = 2(0.4) + 4(0.6)$

$= 0.8 + 2.4$

$= 3.2$ #

2.3 Find $Var(X)$ and $Var(Y)$.

$Var(X) = (1-1.8)^2(0.4) + (2-1.8)^2(0.4) + (3-1.8)^2(0.2)$

$= 0.256 + 0.016 + 0.288$

$= 0.56$ #

$Var(Y) = (2-3.2)^2(0.4) + (4-3.2)^2(0.6)$

$= 0.576 + 0.384$

$= 0.96$ #

2.4 Find $E(X|Y=4)$ and $Var(Y|X=3)$.

$$E(X|Y=4) = \left[1 \cdot \frac{0.3}{0.6}\right] + \left[2 \cdot \frac{0.2}{0.6}\right] + \left[3 \cdot \frac{0.1}{0.6}\right]$$

$$= \frac{1}{2} + \frac{2}{3} + \frac{1}{2} = \frac{5}{3}$$

$$= 1.667 \ \#$$

Find $E(Y|X=3) = \left[2 \cdot \frac{0.1}{0.2}\right] + \left[4 \cdot \frac{0.1}{0.2}\right]$

$$= 3$$

$$Var(Y|X=3) = (2-3)^2\left[\frac{0.1}{0.2}\right] + (4-3)^2\left[\frac{0.1}{0.2}\right]$$

$$= \frac{1}{2} + \frac{1}{2} = 1 \ \#$$

2.5 Define Z as $X - Y$, Find $Var(Z)$.

Z	-3	-2	-1	0	1
$f(Z)$	0.3	0.2	0.2	0.2	0.1

$$E(Z) = (-3)(0.3) + (-2)(0.2) + (-1)(0.2) + (1)(0.1)$$

$$= -1.4 \ \#$$

$$Var(Z) = (-3+1.4)^2(0.3) + (-2+1.4)^2(0.2) + (-1+1.4)^2(0.2) + (0+1.4)^2(0.2) + (1+1.4)^2(0.1) = 0.768 + 0.072 + 0.032 + 0.392 + 0.576 = 1.84 \ \#$$

2.6 Find the $E(E(Y|X))$ and show that $E\big(E(Y|X)\big) = E(Y)$.

From 2.2, $E(Y) = 3.2$

$$E(Y|X) = E(Y|X=1) \cdot P(X=1) + E(Y|X=2) \cdot P(X=2) + E(Y|X=3) \cdot P(X=3)$$

$$\left\{2\left(\frac{0.1}{0.4}\right) + 4\left(\frac{0.3}{0.4}\right)\right\}(0.4) \left\{2\left(\frac{0.2}{0.4}\right) + 4\left(\frac{0.2}{0.4}\right)\right\}(0.4) \left\{2\left(\frac{0.1}{0.2}\right) + 4\left(\frac{0.1}{0.2}\right)\right\}(0.2)$$

$$= (0.5 + 3)(0.4) + (1 + 2)(0.4) + (1 + 2)(0.2)$$

$$= 1.4 + 1.2 + 0.6$$

$$= 3.2 \ \#$$

3. Let X be a continuous random variable, the PDF is given by

$$f(x) = \begin{cases} a + bx^2 & ; 0 \le x \le 1 \\ 0 & ; elsewhere \end{cases}$$

If the expected value $E(X) = \frac{3}{5}$, find the value of a and b.

We know that

(1) $\int_0^1 x\cdot(a+bx^2)\,dx = \frac{3}{5}$

$\int_0^1 ax+bx^3\,dx = \frac{ax^2}{2}+\frac{bx^4}{4}\Big|_0^1$

$= \frac{a}{2}+\frac{b}{4} = \frac{3}{5}$

$2a+b = \frac{12}{5}$

- Replace $b = 3(1-a)$

$2a+[3-3a] = \frac{12}{5}$

$a = \frac{3}{5}$

(2) $\int_0^1 a+bx^2\,dx = 1$

$= ax+\frac{bx^3}{3}\Big|_0^1$

$= a+\frac{b}{3} = 1$

$b = 3(1-a)$

$a = 1-\frac{b}{3}$

Replace $a = 3/5$

$b = 3-3\left(\frac{3}{5}\right) = \frac{6}{5}$

4. Let X and Y be continuous random variables, the PDF is given by

$$f(x,y) = \begin{cases} x^2 + \frac{xy}{k} & ; 0 \le x \le 1, 0 \le y \le 2 \\ 0 & ; elsewhere \end{cases}$$

Answer the following questions.

4.1 Find the value of k.

$\int_0^2\int_0^1 x^2+\frac{xy}{k}\,dx\,dy = 1$

$\int_0^2 \left[\frac{x^3}{3}+\frac{x^2y}{2k}\Big|_0^1\right]dy = 1$

$\int_0^2 \frac{1}{3}+y/2k\,dy = 1$

$\frac{1}{3}y+\frac{y^2}{4k}\Big|_0^2 = 1$

$\frac{2}{3}+\frac{1}{k} = 1$

$\frac{1}{k} = \frac{1}{3}$

$k = 3$ #

4.2 Find $P\left(\frac{1}{4} < X < \frac{1}{2}, Y > 1\right)$.

$\int_1^2\int_{1/4}^{1/2} x^2+\frac{xy}{3}\,dx\,dy = \int_1^2 \left[x^3/3+\frac{x^2y}{6}\Big|_{1/4}^{1/2}\right]dy$

$= \int_1^2 \left\{\left[\frac{1+y}{24}\right]-\left[\frac{1+2y}{192}\right]\right\}dy$

$= \int_1^2 \frac{7+6y}{192}\,dy = \frac{7}{192}y+\frac{6y^2}{384}\Big|_1^2$

$= \left\{\frac{28+24}{384}\right\}-\left\{\frac{14+6}{384}\right\} = \frac{32}{384} = \frac{1}{12}$ #

4.3 Find $P\left(X > \frac{1}{2} | Y = 2\right)$.

$$\int_{1/2}^{1} x^2 + \frac{2x}{3}\, dx = \frac{x^3}{3} + \frac{x^2}{3} \Big|_{1/2}^{1} = \left\{ \frac{1}{3} + \frac{1}{3} \right\} - \left\{ \frac{1}{24} + \frac{1}{12} \right\}$$

$$= \frac{2}{3} - \frac{3}{24} = \frac{13}{24}$$

\#

4.4 Are X and Y independent of each other? Show a proof to your answer.

① Find marginal PDF of $f(x)$ and $f(y)$

- $f(x) = \int_0^2 x^2 + \frac{xy}{3}\, dy$

$= x^2 y + \frac{xy^2}{6} \Big|_0^2$

$= 2x^2 + \frac{2}{3}x$

$f(y) = \int_0^1 x^2 + \frac{xy}{3}\, dx$

$= \frac{x^3}{3} + \frac{x^2 y}{6} \Big|_0^1$

$= \frac{1}{3} + \frac{y}{6} = \frac{2+y}{6}$

② Find $E(X)$ and $E(Y)$

$E(X) = \int_0^1 x \cdot (2x^2 + \frac{2}{3}x)\, dx$

$= \frac{2x^4}{4} + \frac{2}{3} \cdot \frac{x^3}{3} \Big|_0^1$

$= \frac{1}{2} + \frac{2}{9} = \frac{13}{18}$

$E(Y) = \int_0^2 y \left(\frac{2+y}{6}\right) dy$

$= \int_0^2 \frac{2y}{6} + \frac{y^2}{6}\, dy$

$= \frac{2y^2}{12} + \frac{y^3}{18} \Big|_0^2 = \frac{2}{3} + \frac{8}{18} = \frac{20}{18} = \frac{10}{9}$

③ $E(X) \cdot E(Y) = \frac{13}{18} \cdot \frac{10}{9} = \frac{130}{162} = \frac{65}{81}$

④ Find $E(XY)$

$\int_0^2 \int_0^1 xy\left(x^2 + \frac{xy}{3}\right) dx\, dy = \int_0^2 \left\{ \frac{x^4 y}{4} + \frac{x^3 y^2}{9} \Big|_0^1 \right\} dy$

$= \int_0^2 y/4 + y^2/9\, dy$

$= \frac{y^2}{8} + \frac{y^3}{27} \Big|_0^2$

$= \frac{1}{2} + \frac{8}{27} = \frac{43}{54}$

They are not independent!

\#

4.5 Find the correlation coefficient of X and Y (ρ_{XY}).

- $\rho_{xy} = \frac{\text{cov}(x,y)}{\sigma_x \sigma_y}$

- $\text{cov}(x,y) = E(XY) - E(X)E(Y)$

$= \frac{43}{54} - \frac{65}{81} = -\frac{1}{162}$

- $\sigma_x = \sqrt{\text{var}(x)}$, and so does for y

$\text{var}(x) = \int_0^1 (x - E(x))^2 \cdot f(x)\, dx$

$= \int_0^1 (x - \frac{13}{18})^2 \cdot (2x^2 + \frac{2}{3}x)\, dx$

$= 73/1620$; $\sigma_x \approx 0.2123$

$\text{var}(y) = \int_0^2 (y - E(y))^2 \cdot f(y)\, dy$

$= \int_0^2 (y - \frac{10}{9})^2 \cdot \left(\frac{2+y}{6}\right) dy$

$= \frac{26}{81}$; $\sigma_y \approx 0.5666$

- $\rho_{xy} = \frac{-1/162}{(0.2133)(0.5666)} \approx -0.0513$

\#

5. Let $X \sim N(\mu, \sigma^2)$, given that you have two estimators in hand which are

(1) $\bar{X} = \sum_{i=1}^{n} \frac{X_i}{n}$

(2) $\hat{X} = \frac{n}{(n-2)^2} \sum_{i=1}^{n} X_i$

Answer the following questions.

5.1 Show the bias of each estimator and explain which estimator is unbiased.

$$E(\bar{X}) = E\left(\sum \frac{x_i}{n}\right) = E\left[\frac{1}{n}(X_1 + X_2 + \dots X_n)\right] = \frac{1}{n}\left\{E(X_1) + E(X_2) + \dots + E(X_n)\right\} = \frac{n\mu}{n} = \mu$$

bias $(\bar{X}) = 0$ #

$$E(\hat{X}) = E\left[\frac{n}{(n-2)^2}\sum X_i\right] = \frac{n}{(n-2)^2} E\left(\sum X_i\right) = \frac{n^2\mu}{(n-2)^2}$$

biased ! #

5.2 Which estimator is more efficient? Show the answer with a proof.

$$var(\bar{X}) = \frac{1}{n^2} var(X_1 + X_2 + \dots + X_n) = \frac{1}{n^2} \cdot n\sigma^2 = \frac{\sigma^2}{n}$$

$$var(\hat{X}) = var\left[\frac{n}{(n-2)^2}\sum X_i\right] = \frac{n^2}{(n-2)^4} \cdot n\sigma^2 = \frac{n^3\sigma^2}{(n-2)^4}$$

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assume:

$$\frac{\sigma^2}{n} < \frac{n^3\sigma^2}{(n-2)^4}$$

$$\cancel{\sigma^2} < \left[\frac{n}{(n-2)}\right]^4 \cancel{\sigma^2}$$

take $\sqrt[4]{\ }$ both sides:

$$1 < \frac{n}{n-2}$$

is true when $n > 2$

$\therefore var(\bar{X}) < var(\hat{X})$ when $n > 2$

5.3 Which estimator that you pick to represent the population and why?

$\bar{X}$ because it is unbiased and when sample more then 2 time, $\bar{X}$ is more efficient.