Quiz 5

1. Find

$$\int (1-x)\sin(\pi x)\ dx.$$

Solution: Integration by parts:

- Let $u = 1 - x$ and $dv = \sin(\pi x)dx$.
- Then $du = -dx$ and
- $v = \int \sin(\pi x)\ dx = \int \sin(\pi x)\ \frac{1}{\pi} d(\pi x) = -\frac{\cos(\pi x)}{\pi} + C$. By setting $C = 0 \ \Rightarrow \ v = -\frac{\cos(\pi x)}{\pi}$.

$$\begin{aligned}
\int (1-x)\sin(\pi x)\ dx &= uv - \int v\ du \\
&= (1-x)\left[-\frac{\cos(\pi x)}{\pi}\right] - \int \left[-\frac{\cos(\pi x)}{\pi}\right](-1)dx \\
&= (1-x)\left[-\frac{\cos(\pi x)}{\pi}\right] - \frac{1}{\pi}\int \cos(\pi x)\ dx \\
&= (1-x)\left[-\frac{\cos(\pi x)}{\pi}\right] - \frac{1}{\pi}\int \cos(\pi x)\ \frac{1}{\pi} d(\pi x) \\
&= (1-x)\left[-\frac{\cos(\pi x)}{\pi}\right] - \frac{1}{\pi^2}\int \cos(\pi x)\ d(\pi x) \\
&= -\left[\frac{(1-x)\cos(\pi x)}{\pi}\right] - \frac{1}{\pi^2}\sin(\pi x) + C
\end{aligned}$$

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