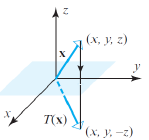
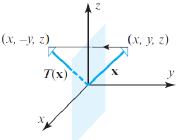
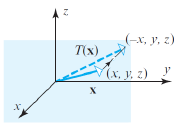
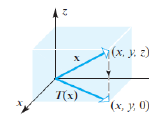
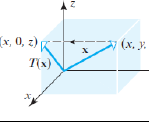
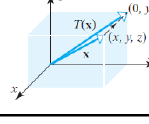


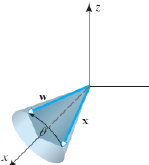
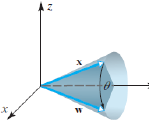
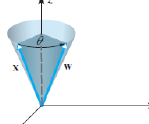
Formula Sheet

Definition 0.1. A vector space (over $\mathbb{R}$) consists of a set V along with two operations: an addition, '+', and a scalar multiplication, ' $\cdot$ ', subject to the conditions that for all vectors $u, v, w, u \in V$ and all scalars $r, s \in \mathbb{R}$:

- (1) the set V is closed under vector addition, that is, $u + v \in V$
- (2) vector addition is commutative, $u + v = v + u$
- (3) vector addition is associative, $(u + v) + w = u + (v + w)$
- (4) there is a zero vector $0 \in V$ such that $u + 0 = u$ for all $u \in V$
- (5) each $u \in V$ has an additive inverse $w \in V$ such that $u + w = 0$
- (6) the set V is closed under scalar multiplication, that is, $r \cdot u \in V$
- (7) addition of scalars distributes over scalar multiplication, $(r + s) \cdot u = r \cdot u + s \cdot u$
- (8) scalar multiplication distributes over vector addition, $r \cdot (u + v) = r \cdot u + r \cdot v$
- (9) ordinary multiplication of scalars associates with scalar multiplication, $(rs) \cdot u = r \cdot (s \cdot u)$
- (10) multiplication by the scalar 1 is the identity operation, $1 \cdot u = u$.

Operator	Illustration	Images of e_1, e_2, e_3	Standard Matrix
Reflection about the xy -plane $T(x, y, z) = (x, y, -z)$		$T(e_1) = T(1, 0, 0) = (1, 0, 0)$ $T(e_2) = T(0, 1, 0) = (0, 1, 0)$ $T(e_3) = T(0, 0, 1) = (0, 0, -1)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
Reflection about the xz -plane $T(x, y, z) = (x, -y, z)$		$T(e_1) = T(1, 0, 0) = (1, 0, 0)$ $T(e_2) = T(0, 1, 0) = (0, -1, 0)$ $T(e_3) = T(0, 0, 1) = (0, 0, 1)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Reflection about the yz -plane $T(x, y, z) = (-x, y, z)$		$T(e_1) = T(1, 0, 0) = (-1, 0, 0)$ $T(e_2) = T(0, 1, 0) = (0, 1, 0)$ $T(e_3) = T(0, 0, 1) = (0, 0, 1)$	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Operator	Illustration	Images of e_1, e_2, e_3	Standard Matrix
Orthogonal projection onto the xy -plane $T(x, y, z) = (x, y, 0)$		$T(e_1) = T(1, 0, 0) = (1, 0, 0)$ $T(e_2) = T(0, 1, 0) = (0, 1, 0)$ $T(e_3) = T(0, 0, 1) = (0, 0, 0)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Orthogonal projection onto the xz -plane $T(x, y, z) = (x, 0, z)$		$T(e_1) = T(1, 0, 0) = (1, 0, 0)$ $T(e_2) = T(0, 1, 0) = (0, 0, 0)$ $T(e_3) = T(0, 0, 1) = (0, 0, 1)$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
Orthogonal projection onto the yz -plane $T(x, y, z) = (0, y, z)$		$T(e_1) = T(1, 0, 0) = (0, 0, 0)$ $T(e_2) = T(0, 1, 0) = (0, 1, 0)$ $T(e_3) = T(0, 0, 1) = (0, 0, 1)$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Operator	Illustration	Rotation Equations	Standard Matrix
Counterclockwise rotation about the positive x -axis through an angle θ		$w_1 = x$ $w_2 = y \cos \theta - z \sin \theta$ $w_3 = y \sin \theta + z \cos \theta$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix}$
Counterclockwise rotation about the positive y -axis through an angle θ		$w_1 = x \cos \theta + z \sin \theta$ $w_2 = y$ $w_3 = -x \sin \theta + z \cos \theta$	$\begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix}$
Counterclockwise rotation about the positive z -axis through an angle θ		$w_1 = x \cos \theta - y \sin \theta$ $w_2 = x \sin \theta + y \cos \theta$ $w_3 = z$	$\begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$