

# **EE312 Macroeconomic Theory**

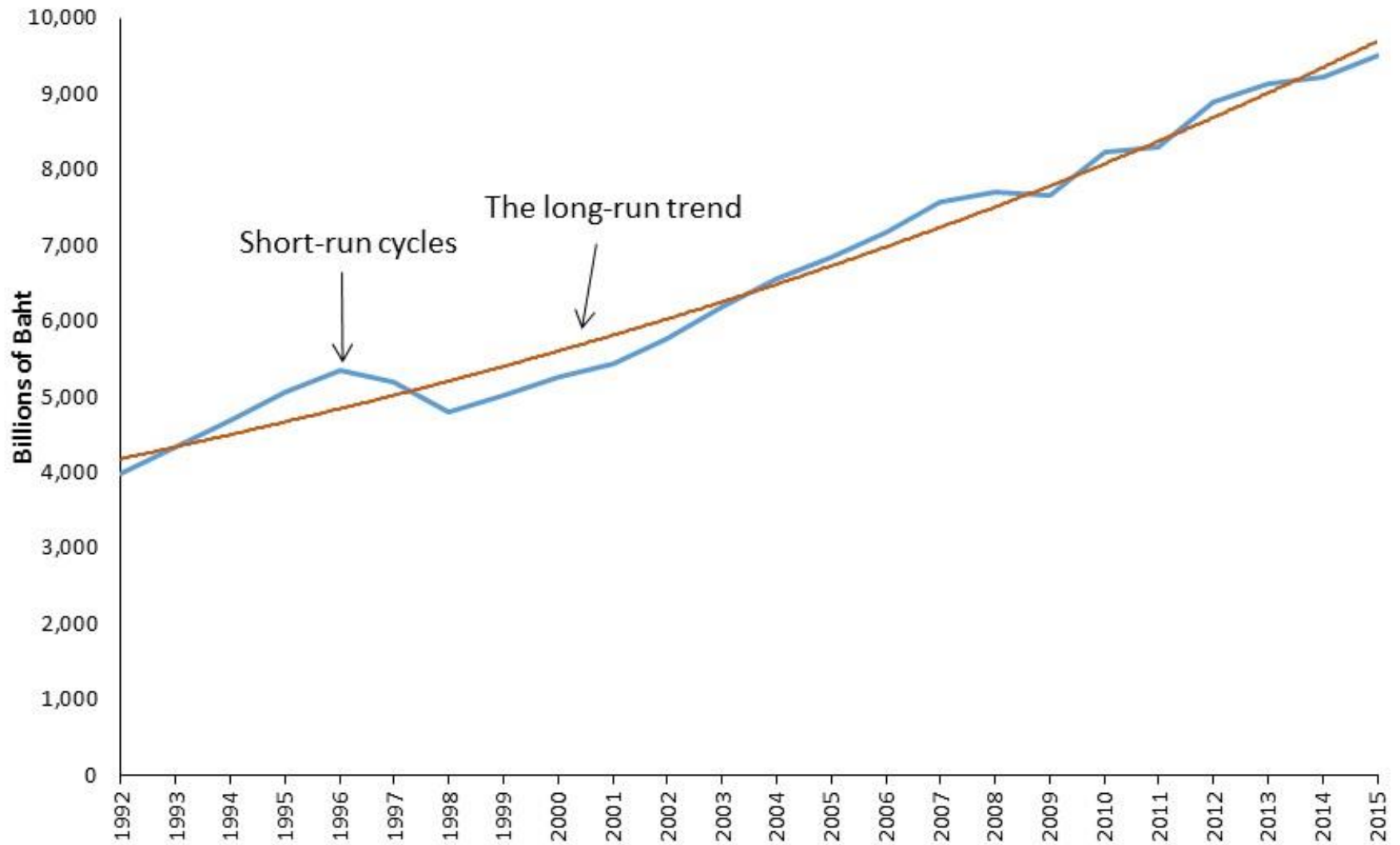
Long-term Economic Growth

The Solow growth model

# Importance of growth

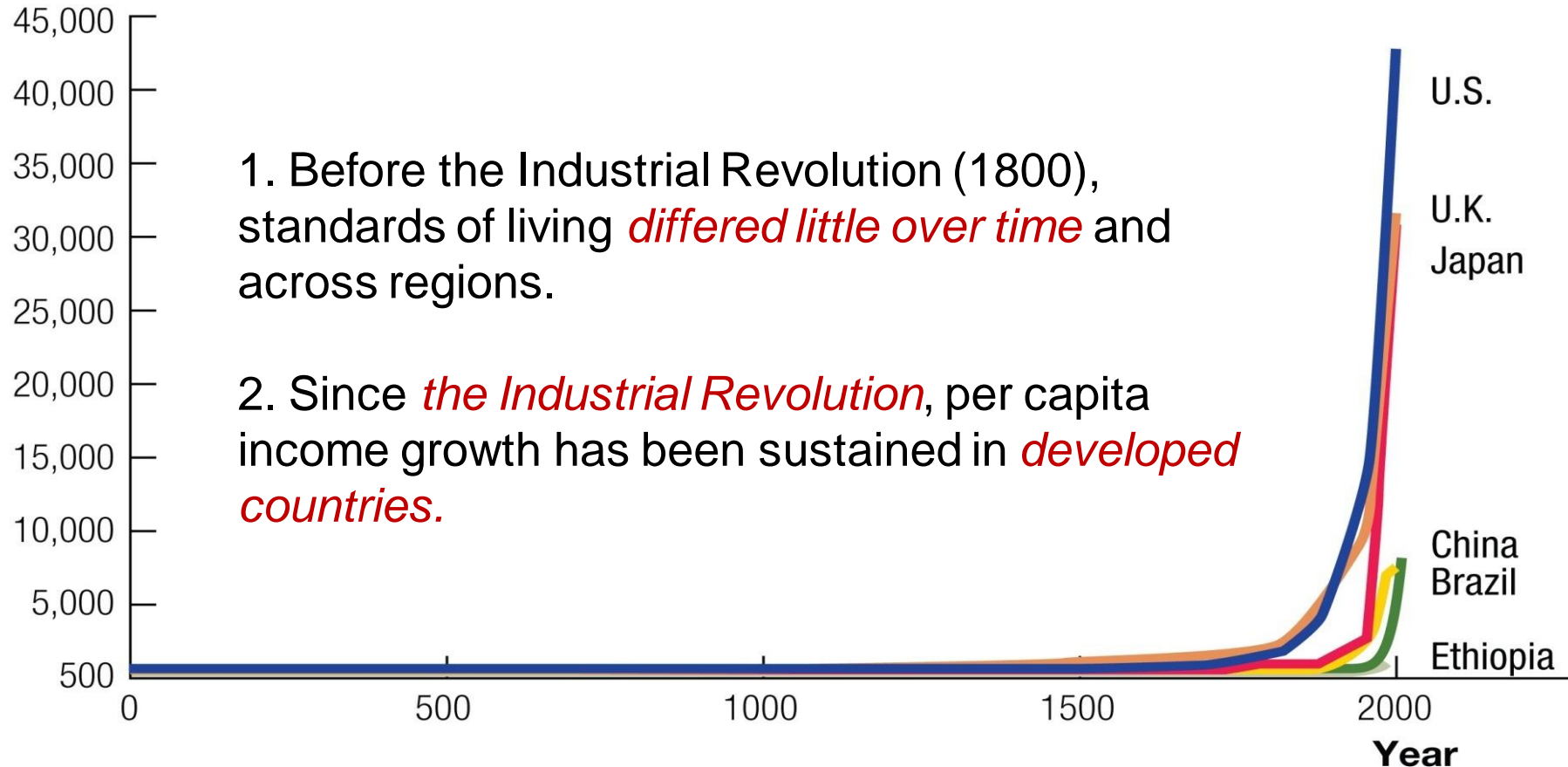
- The standards of living in the long term depend on **economic growth**.
  - Short-run fluctuations tend to cancel out in the long run.
- What determines economic growth?
  - **Models of economic growth**.
    - The Solow growth model.
    - Endogenous growth models.

## Thailand's GDP (CVM2002)



# Facts about growth

Per capita GDP  
(2005 dollars)

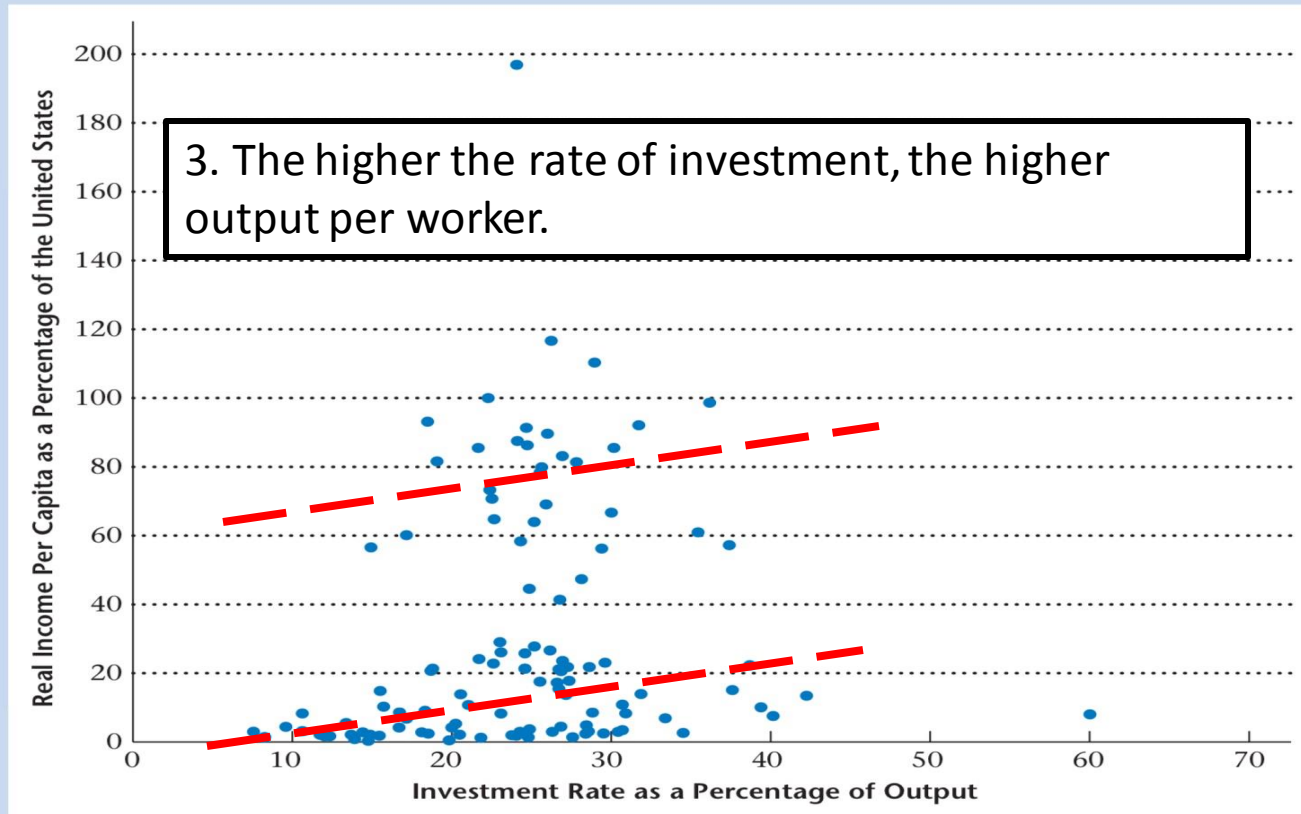


# Facts about growth

**Figure 7.2** Real Income Per Capita vs. Investment Rate

The figure shows a positive correlation across the countries of the world, between the output per capita and the investment rate.

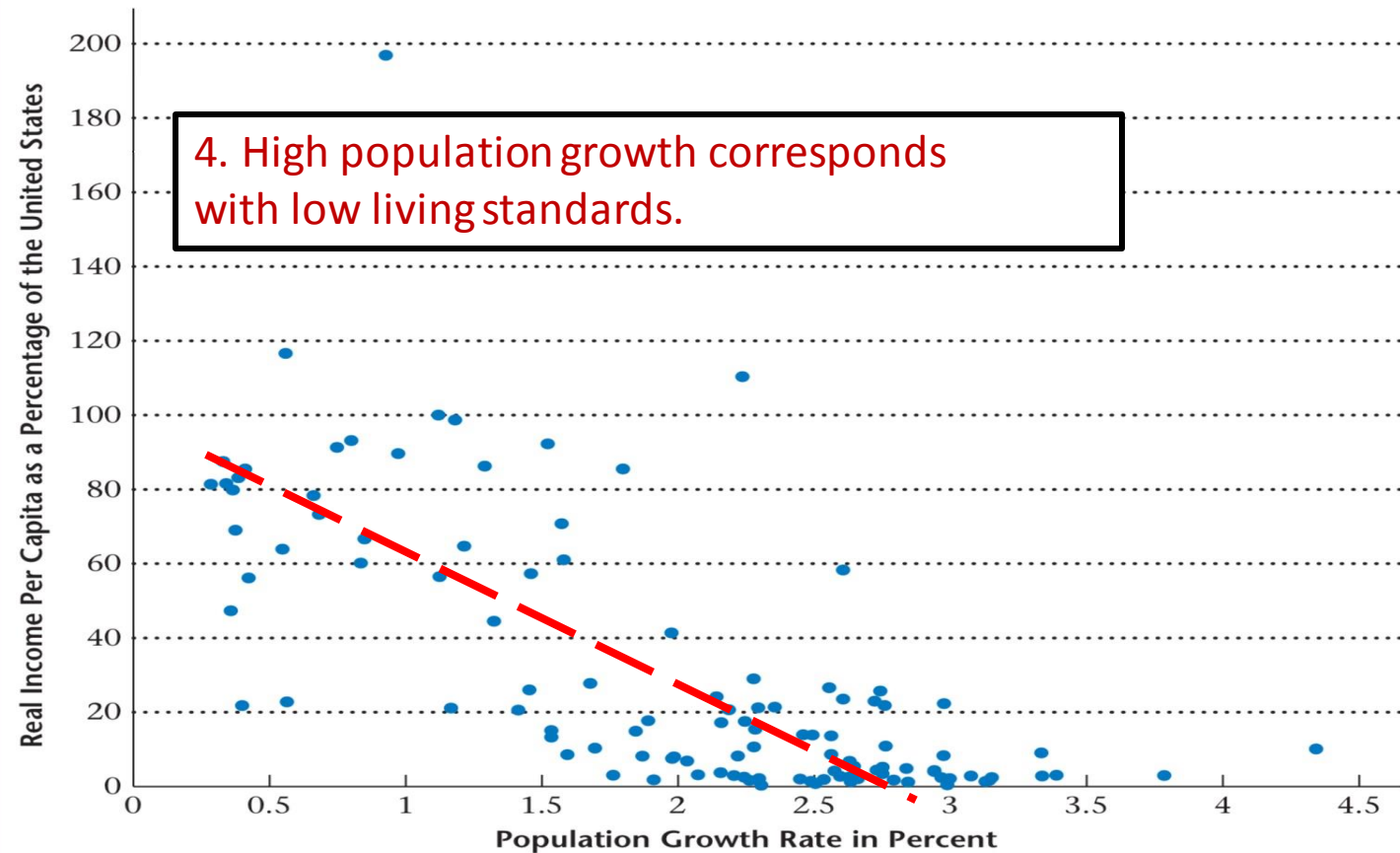
Source: A. Heston, R. Summers, and B. Aten, *Penn World Table Version 7.0*, Center for International Comparisons of Production, Income, and Prices at the University of Pennsylvania, May 2011.



# Facts about growth

Figure 7.3 Real Per Capita Income vs. the Population Growth Rate

Across the countries in the world, real per capita income and the population growth rate are negatively correlated.



# Facts about growth

5. One upset outcome of growth development process: Huge international differences in living standards **increasingly and persistently widen** between rich and poor countries



Poverty in India

Affluent society in developed countries.

# World Bank classification 2016

- **Gross National Income (GNI) per capita**  
(Atlas method).
  - High income (79) > \$12,235
  - Upper middle income (55) \$3,956 - \$12,235
  - Lower middle income (52) \$1,006 - \$3,955
  - Low income (31) < \$1,006
  - **World average** \$10,302

- **High income:** North America, Western Europe, East Asia, oil-rich Middle East.
- **Upper middle income:** South America, Eastern Europe, Russia, South-East Asia.
- **Lower middle income:** South America, Central Asia, Africa.
- **Low income:** Central and East Africa

# Rich and poor in Asia

Japan	38,000	Singapore	51,880
<b>S. Korea</b>	27,600	<b>Malaysia</b>	9,850
Hong Kong	43,420	<b>Thailand</b>	<b>5,640</b>
China	8,260	Philippines	3,580
Bhutan	2,510	Indonesia	3,400
India	1,680	Vietnam	2,050
Pakistan	1,510	Lao	2,150
Bangladesh	1,330	Myanmar	1,190
Nepal	730	Cambodia	1,140

# Facts about growth

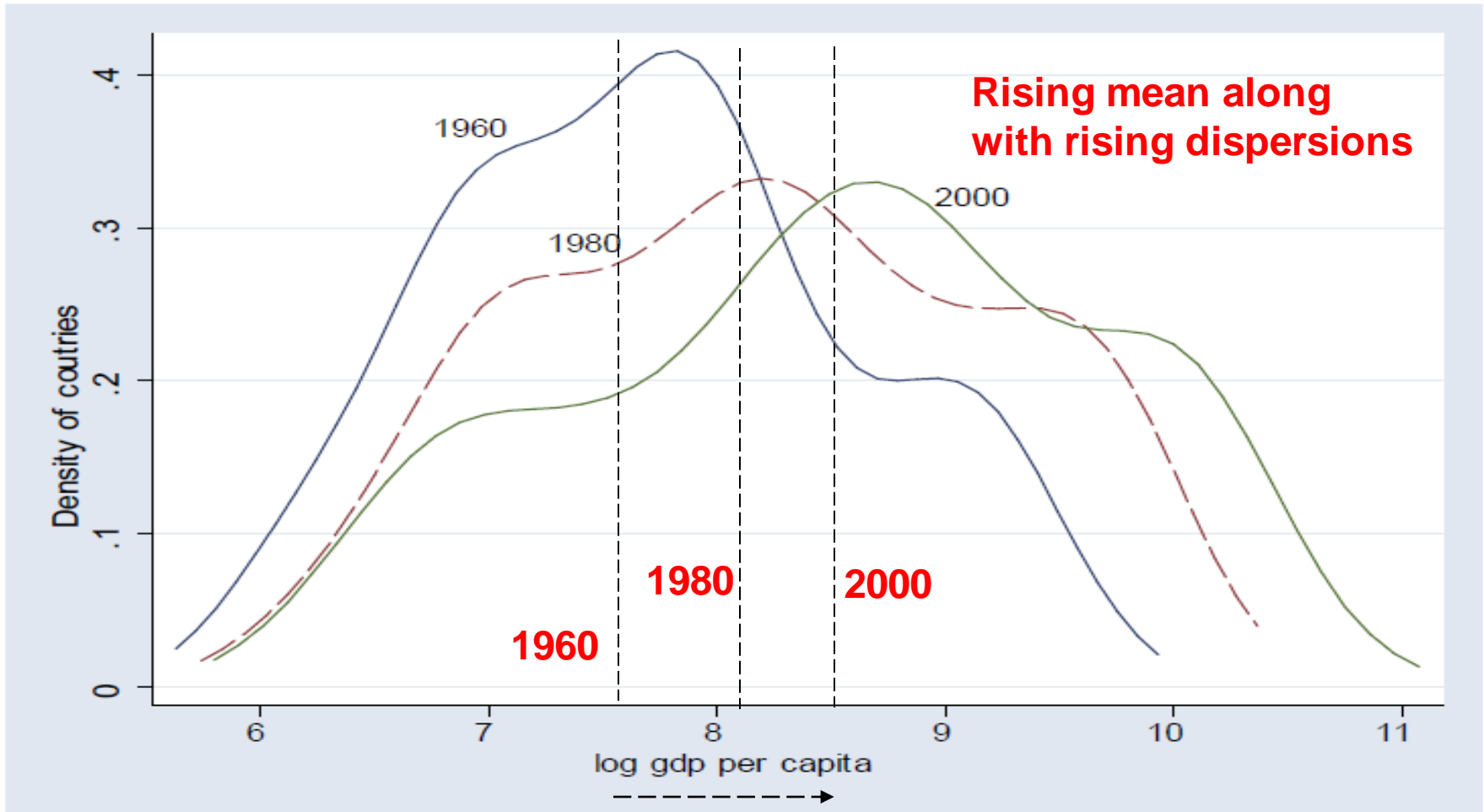


Figure: Estimates of the distribution of countries according to log GDP per capita (PPP-adjusted) in 1960, 1980 and 2000.

# Facts about growth

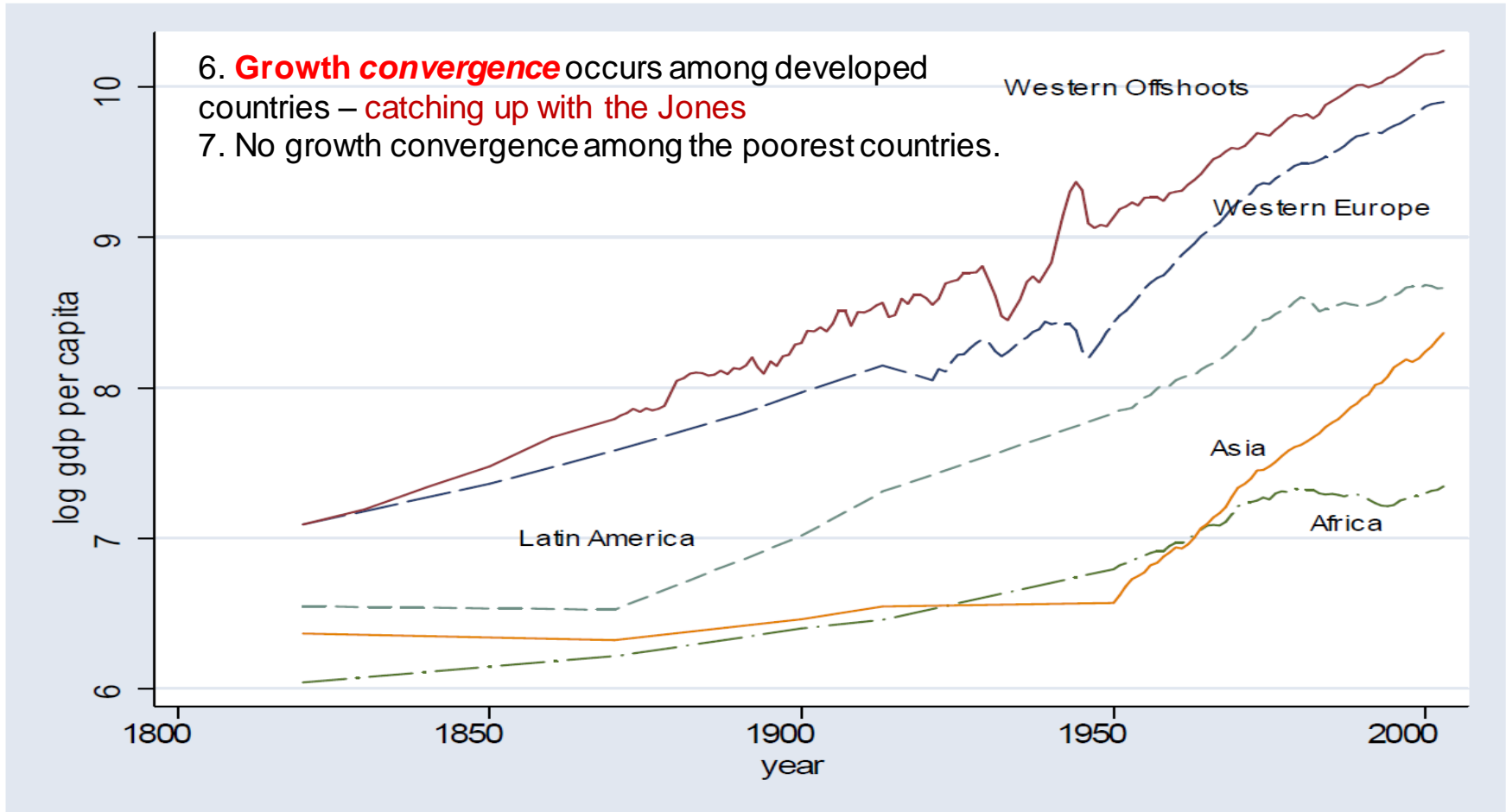


Figure: Evolution of GDP per capita 1820-2000.

# Facts about growth

Seemingly, slower growth rate as income per capita increases

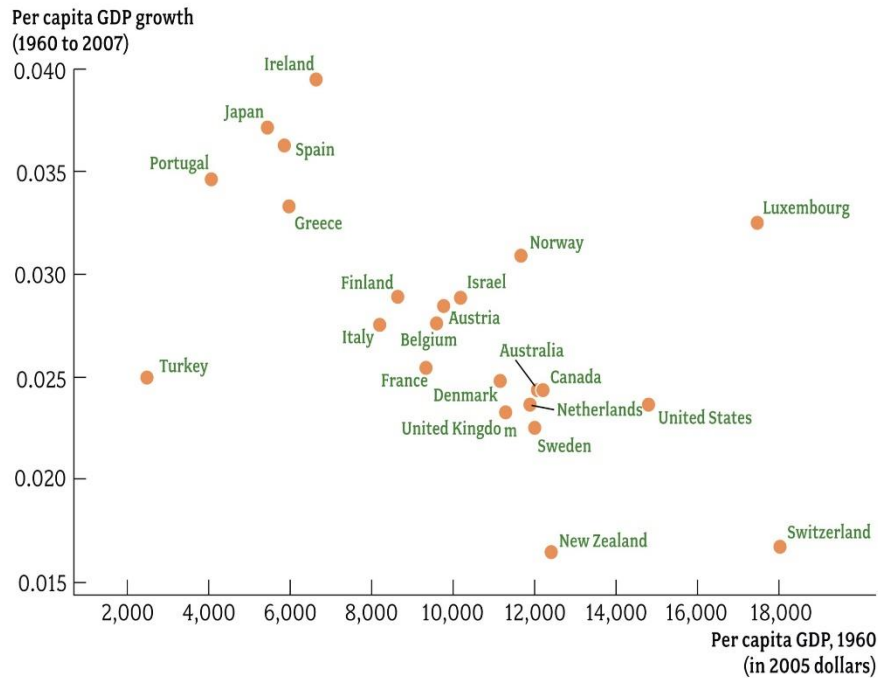


FIGURE 5.8 Growth Rates in the OECD, 1960–2007

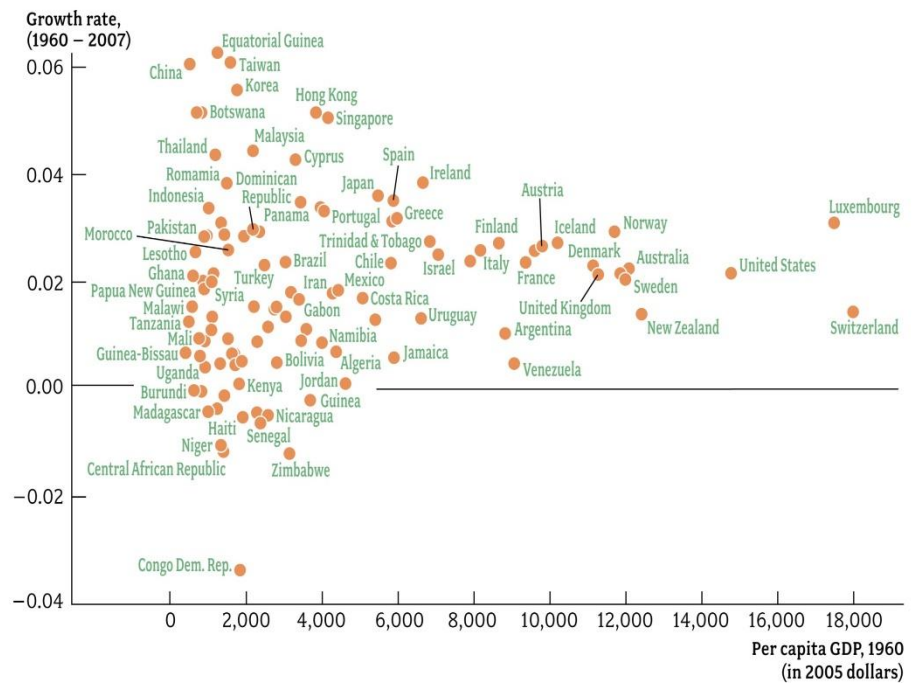


FIGURE 5.9 Growth Rates around the World, 1960–2007

Growth negatively relates to the level of income per capita!

**Implications:** Convergence!

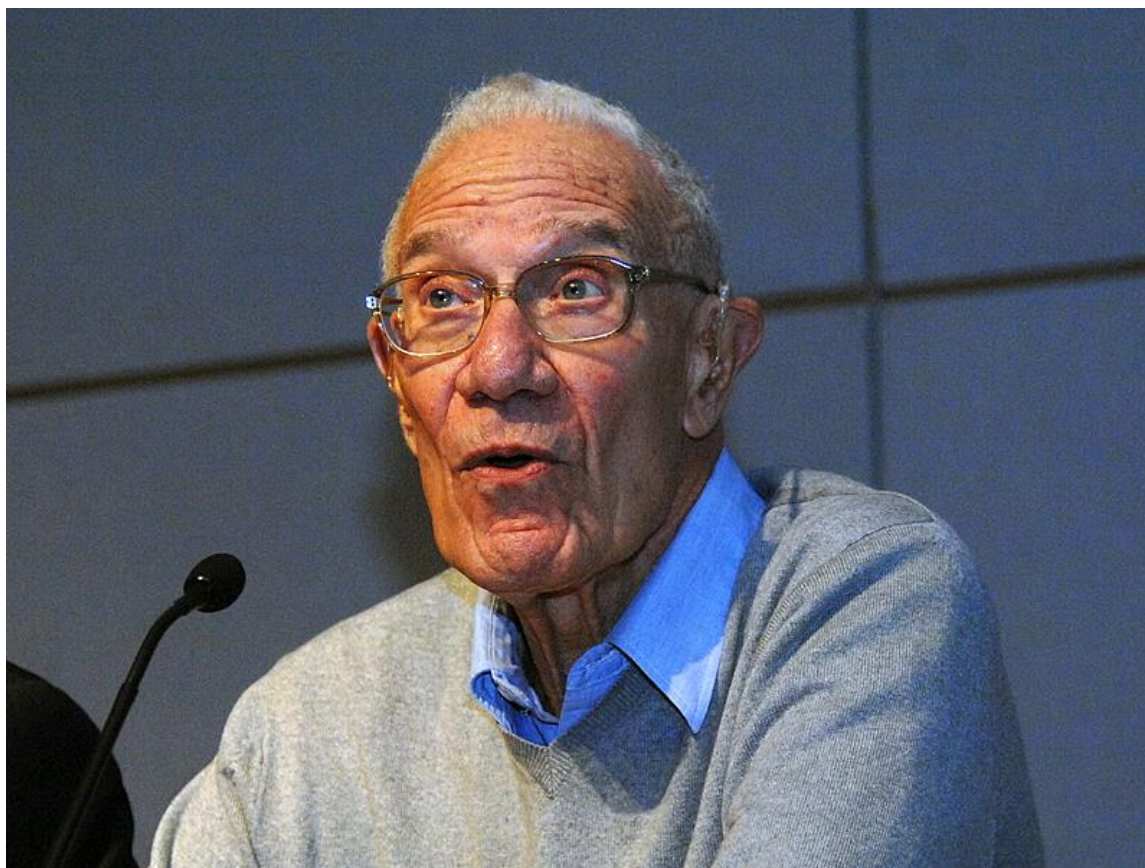
**Data?** -> does not seem to suggest so! No, (absolute) converge

# Questions

- *Positive questions*
  - What is the growth process?
  - Do we expect a convergence in growth?
  - Why don't countries converge to the same growth outcome?
  - What causes income disparities at the global level?
- *Normative questions*
  - Growth policies → closing the gap!

# Growth models

- **Solow growth:** sustainable growth based on technological progress.
  - **Exogenous growth:** *technology is determined outside the model.*
  - Growth convergence among countries.
- **Endogenous growth:** sustainable growth based on human capital.
  - Growth engine is **endogenous**.
  - No certainty in growth convergence.



**Robert M. Solow** (b.1924),  
Massachusetts Institute of Technology,  
Nobel Prize 1987

# The Solow growth model

- The basis of all modern theories of growth.
- Long-term economic growth depends on one single factor --- **technological progress**.
  - Rising **total factor productivity (z)**.
  - Sustained improvement in living standards (real per capita income or output per worker).

# Population growth

- Assume population grows exogenously at a constant rate.
- $N$  = population in the current period.
- $N'$  = population in the future period.
- $n > -1$ ; rate of population growth.

$$N' = (1 + n)N$$

# Consumers

- Consumers = population = workers.
- Consumers supply labor in production.
- Consumers receive real output ( $Y$ ) as (wage and dividend) income.
  - Spend on consumption goods ( $C$ ) and save a constant fraction ( $s$ ) of  $Y$  as savings ( $S$ ).

$$Y = C + S; \quad S = sY$$

$$C = (1 - s)Y$$

# The representative firm

- The firm produces output using current capital stock ( $K$ ) and current labor input ( $N$ ).
- Assuming **constant returns to scale**.

$$Y = zF(K, N)$$

$$\frac{Y}{N} = zF\left(\frac{K}{N}, 1\right)$$

# Per worker production function

*Let  $y = \frac{Y}{N} = \text{output per worker}$*

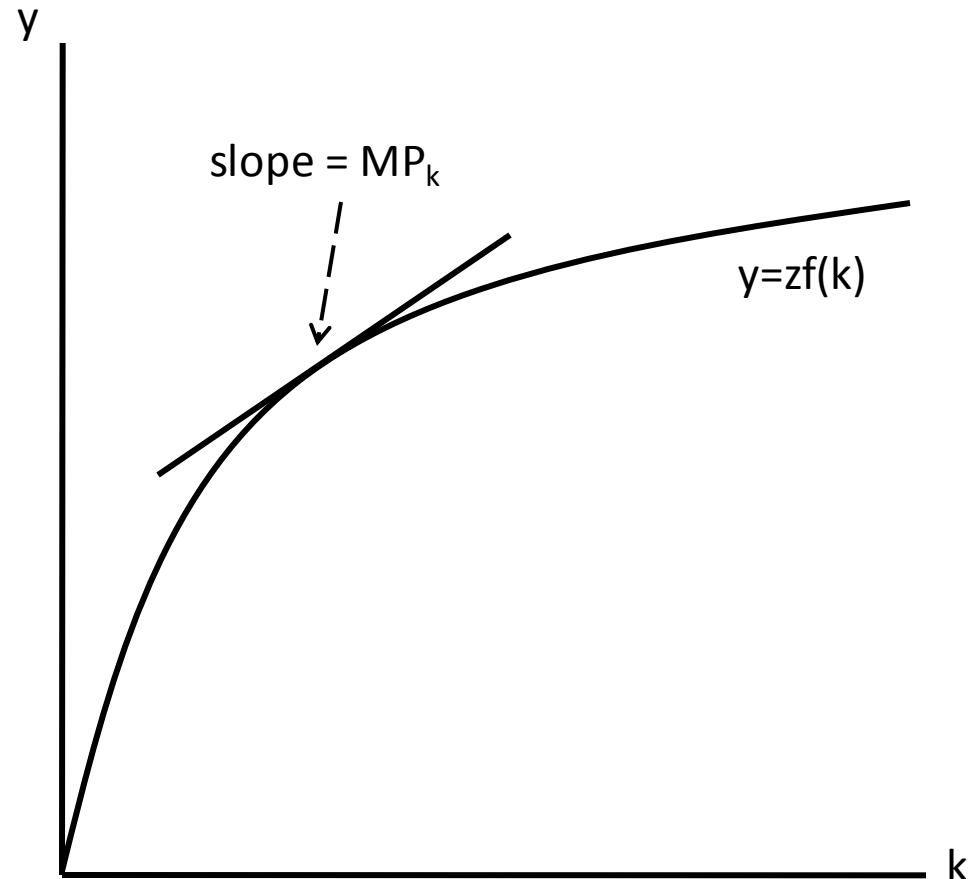
*$k = \frac{K}{N} = \text{capital per worker}$*

$$f(k) = zF\left(\frac{K}{N}, 1\right)$$

$$y = zf(k)$$

# Marginal product of k

- Output per worker ( $y$ ) **increases at a decreasing rate** as capital per worker ( $k$ ) rises.
- Slope is the marginal product of  $k$ .



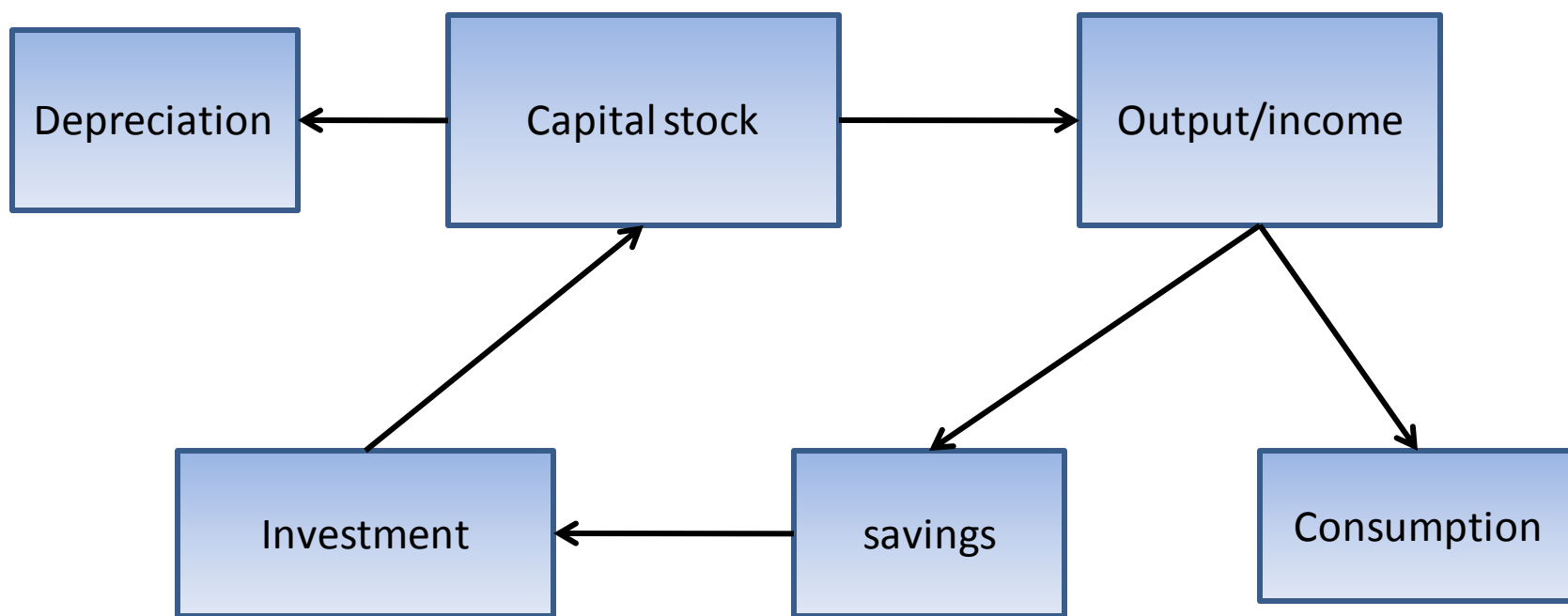
# Growth of capital stock

## Capital accumulation

- Assume capital wears out over time at the rate of  $d$  (or depreciation).
  - where  $0 < d < 1$ .
- $I$  = investment = addition to capital stock.
- $K'$  = capital stock in the future period.

$$K' = (1 - d)K + I$$

# The working of growth process: capital accumulation drives output



# Solution of model: Equilibrium output

- At equilibrium, **savings equals investment** so that output consists of consumption and investment.

$$S = I$$

$$S = Y - C$$

$$Y = C + I$$

# Equilibrium condition

- The future capital stock is current capital stock deducted by depreciation and added by investment (= savings).

$$Y = C + I$$

$$C = (1 - s)Y$$

$$I = K' - (1 - d)K$$

- Substitute C and I in the Y equation.

# Per worker formulation

$$Y = (1 - s)Y + K' - (1 - d)K$$

*rearrange the terms:*

$$K' = sY + (1 - d)K$$

*but*  $Y = zF(K, N)$

*so*  $K' = szF(K, N) + (1 - d)K$

*divide it by N :*

$$\frac{K'}{N} = sz \frac{F(K, N)}{N} + (1 - d) \frac{K}{N}$$

# Future capital per worker function

$$\frac{K'}{N} \frac{N'}{N'} = szF\left(\frac{K}{N}, 1\right) + (1-d) \frac{K}{N}$$

where  $k' = \frac{K'}{N'}$  and  $\frac{N'}{N} = (1+n)$

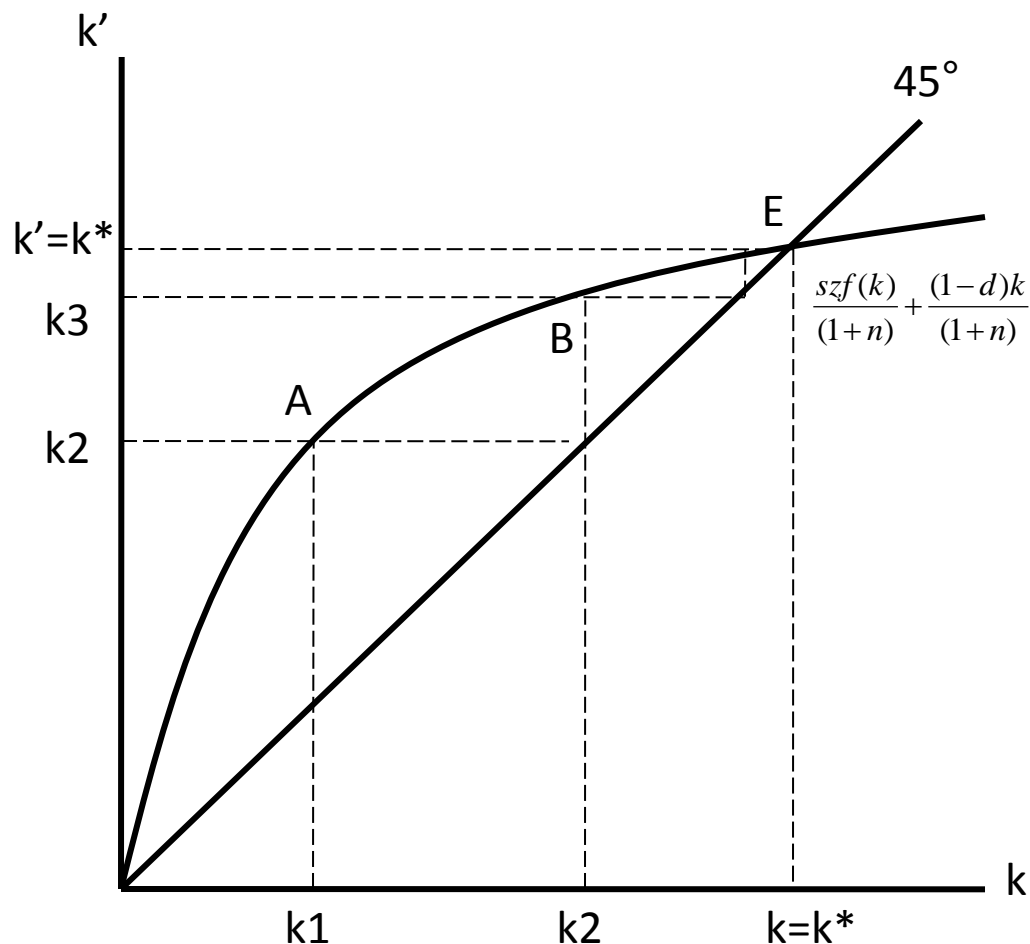
$$k'(1+n) = szf(k) + (1-d)k$$

$$k' = \frac{szf(k)}{(1+n)} + \frac{(1-d)k}{(1+n)}$$

- Future  $k'$  as a function of current  $k$ .

# The steady-state capital per worker

- At A,  $k_2 > k_1$ ;  $k$  is growing.
- At B,  $k_3 > k_2$ ;  $k$  is growing.
- $k = k^*$ ; steady-state capital per worker.



# Diminishing returns on $k$

- At  $E$ ,  $k = k' = k^*$  so that  $k^*$  is steady.
  - To the left of  $k^*$ ,  $k' > k$  so that  $k$  is increasing.
  - To the right of  $k^*$ ,  $k' < k$  so that  $k$  is decreasing.
- As  $k$  is increasing,  $MP_k$  is falling so that  **$y$  is increasing at a decreasing rate.**
- Finally, **investment** (or new capital) is just sufficient to keep up with **population growth and depreciation**, so that  $k$  (and  $y$ ) is stagnant.

# Steady-state aggregates

- With  $k^*$  at the steady state,  $y^*$ ,  $c^*$  and  $szf(k^*)$  are all at **the steady-state**.
  - No further improvement in output per worker ( $y$ ).
- Given population growth ( $n$ ), total factor productivity ( $z$ ) and the savings rate ( $s$ ), the steady-state growth rate is 'n' for aggregate quantities:
  - Capital stock ( $K$ ) and output ( $Y$ );
  - Consumption ( $C$ ), savings ( $S$ ) and investment ( $I$ ).

# Analysis of the steady-state

$$k^* = \frac{szf(k^*)}{(1+n)} + \frac{(1-d)k^*}{(1+n)}$$

Multiplying  $k^*$  by  $(1+n)$

$$szf(k^*) = (n+d)k^*$$

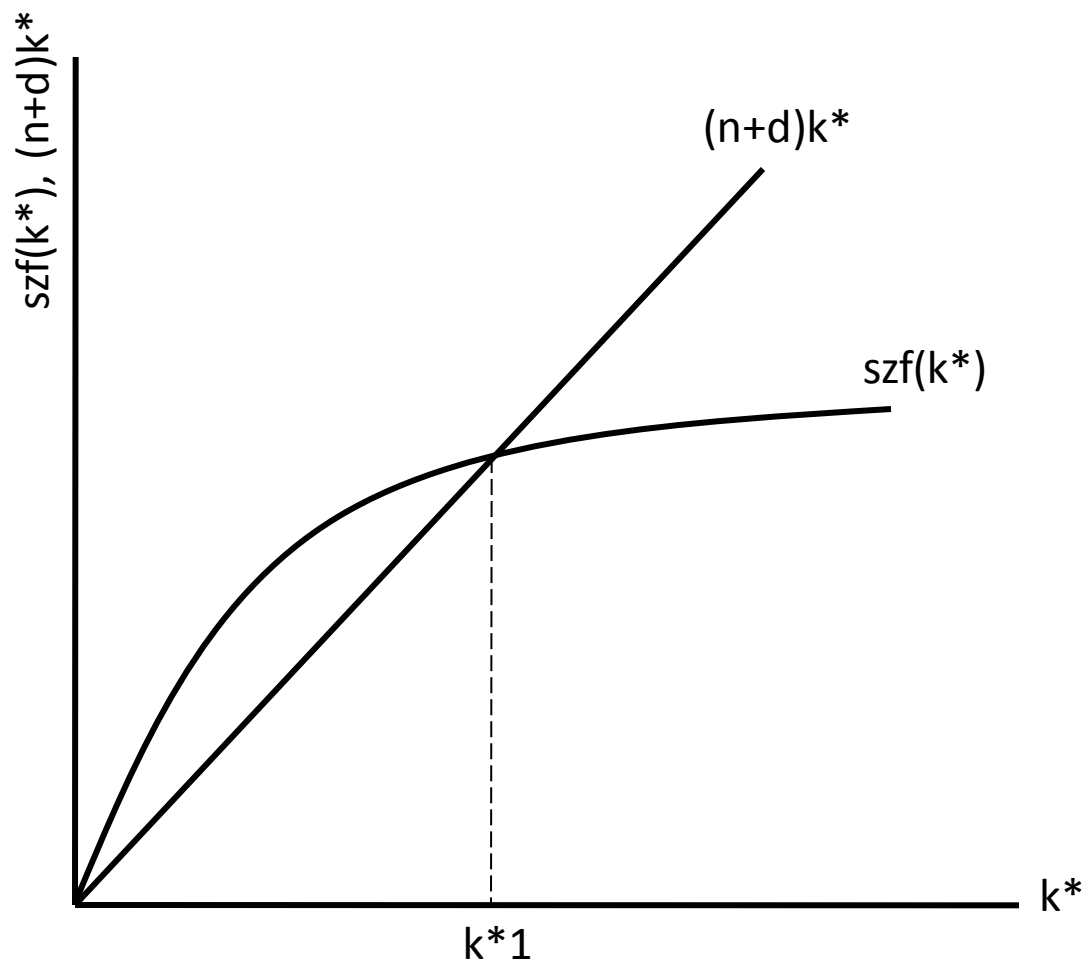
- Or steady-state savings = steady-state investment.

$$szf(k^*) = (n + d)k^*$$

- $szf(k^*)$  = savings per worker;
- $(n+d)k^*$  = investment per worker needed to keep up with population growth and depreciation.
- At  $k^*$ , the capital stock is still growing, but just sufficient to equip each worker with the same  $k$  and depreciation (so  $k^*$  is steady).
  - **‘Capital widening’**: growing  $K$  just to keep the steady  $k$  and  $y$ .

# Determination of steady-state $k^*$

- $szf(k^*)$  is concave due to  $zf(k^*)$ .
- $(n+d)k^*$  has the slope =  $(n+d)$ .



# Policy experiments

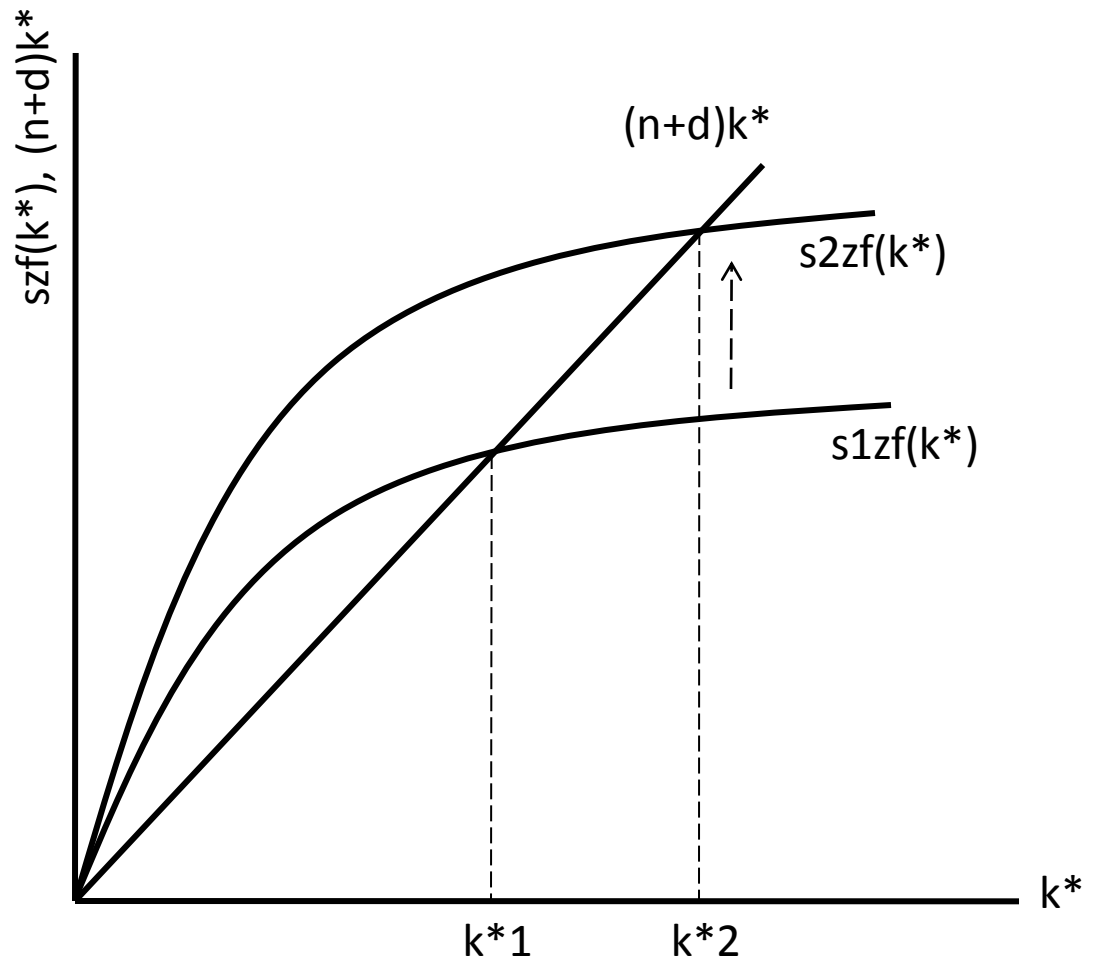
- Change in saving rate ( $s$ )
- Change in population growth ( $n$ )
- Change in the level of technology ( $z$ )

# Effect of an increase in $s$

- Savings rate may increase due to changes in consumers' propensity or government policy.
- Assume a permanent increase in  $s$ :
  - $szf(k^*)$  rotates upwards.
  - Higher steady-state  $k^*$  and  $y^*$  (on a different 'growth path').
  - Higher growth of  $K$  and  $Y$  is transitional.
  - Convergence to the same steady-state growth rate of ' $n$ '.

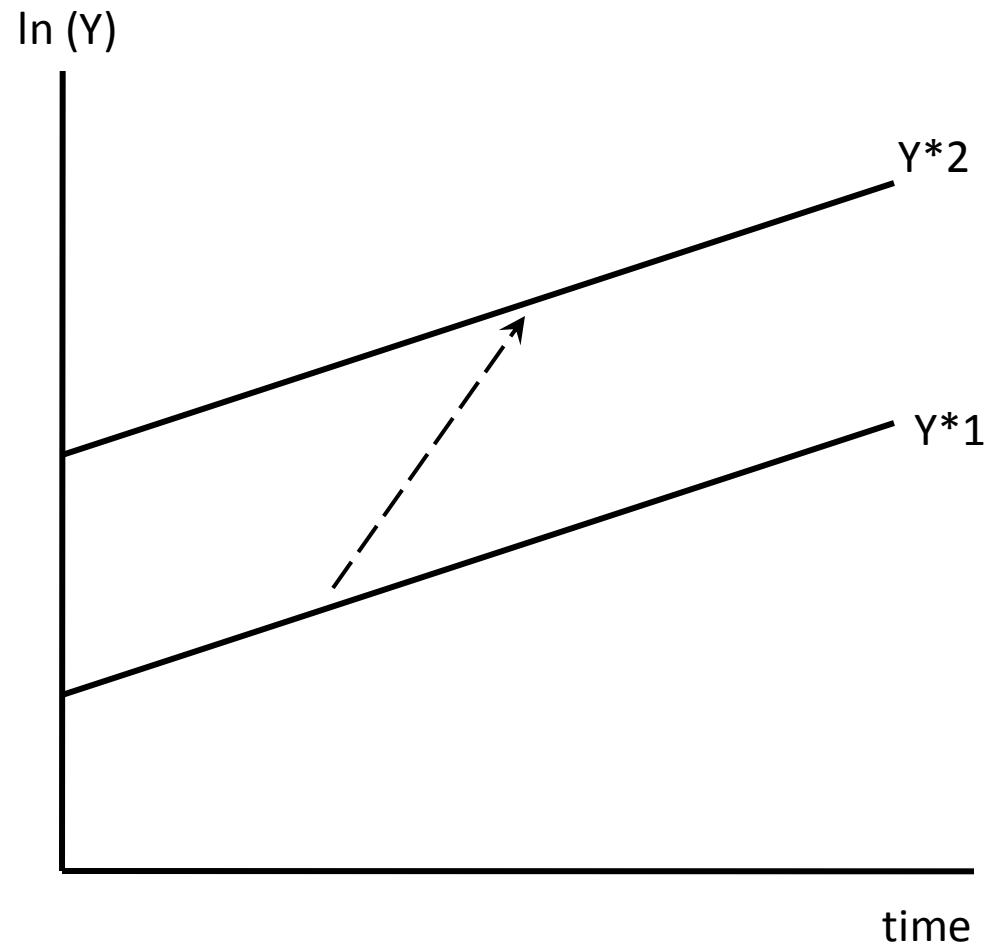
# A rise in $s$ raises $k^*$ .

- Higher savings rate results in a higher  $k^*$  and  $y^*$ .



# Temporary gain in growth rate

- K and Y move to new 'growth paths'.
- Higher growth rates of K and Y are transitional, converging to  $n$ .



# Steady-state consumption per worker

- A broad measure of aggregate welfare.

$$y^* = zf(k^*)$$

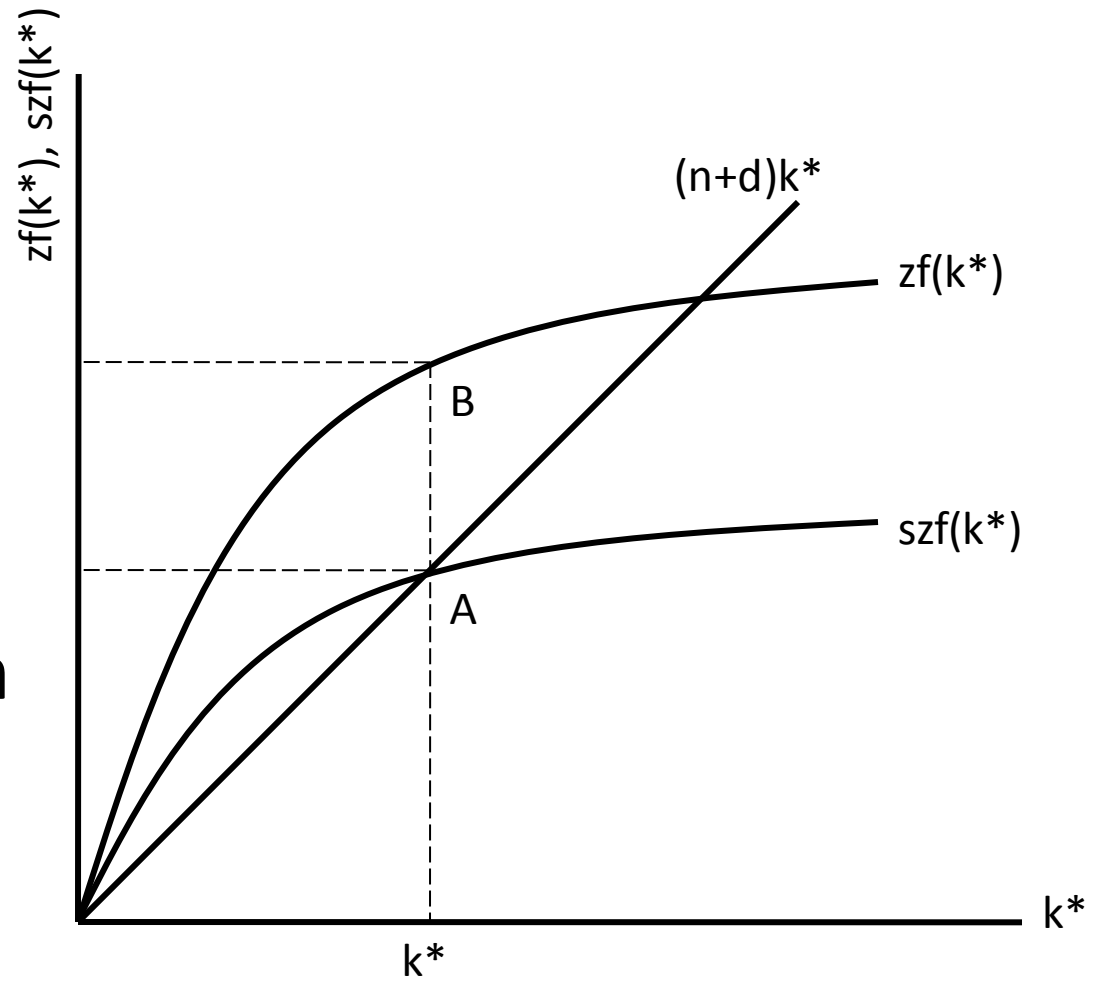
$$\frac{S}{N} = szf(k^*)$$

$$c^* = zf(k^*) - szf(k^*)$$

$$c^* = zf(k^*) - (n + d)k^*$$

$$c^* = (1 - s)zf(k^*)$$

- $AB = c^*$ .
- Each savings rate is associated with a value of steady-state consumption per worker ( $c^{**}$ ).



# Maximized $c^{**}$

$$c^* = zf(k^*) - (n + d)k^*$$

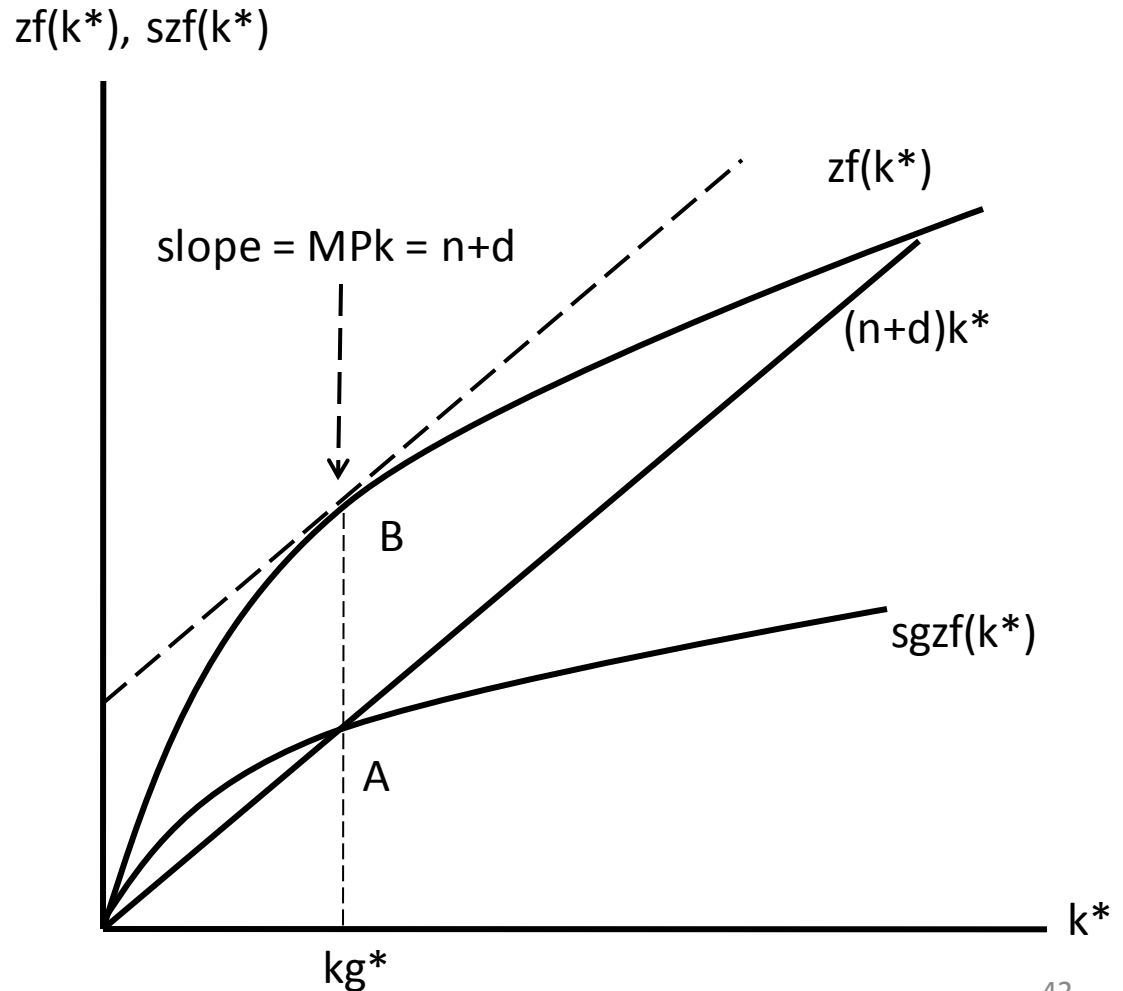
$$\text{Set } \frac{dc^*}{dk^*} = \frac{d(zf(k^*))}{dk^*} - (n + d) = 0$$

$$\frac{d(zf(k^*))}{dk^*} = n + d$$

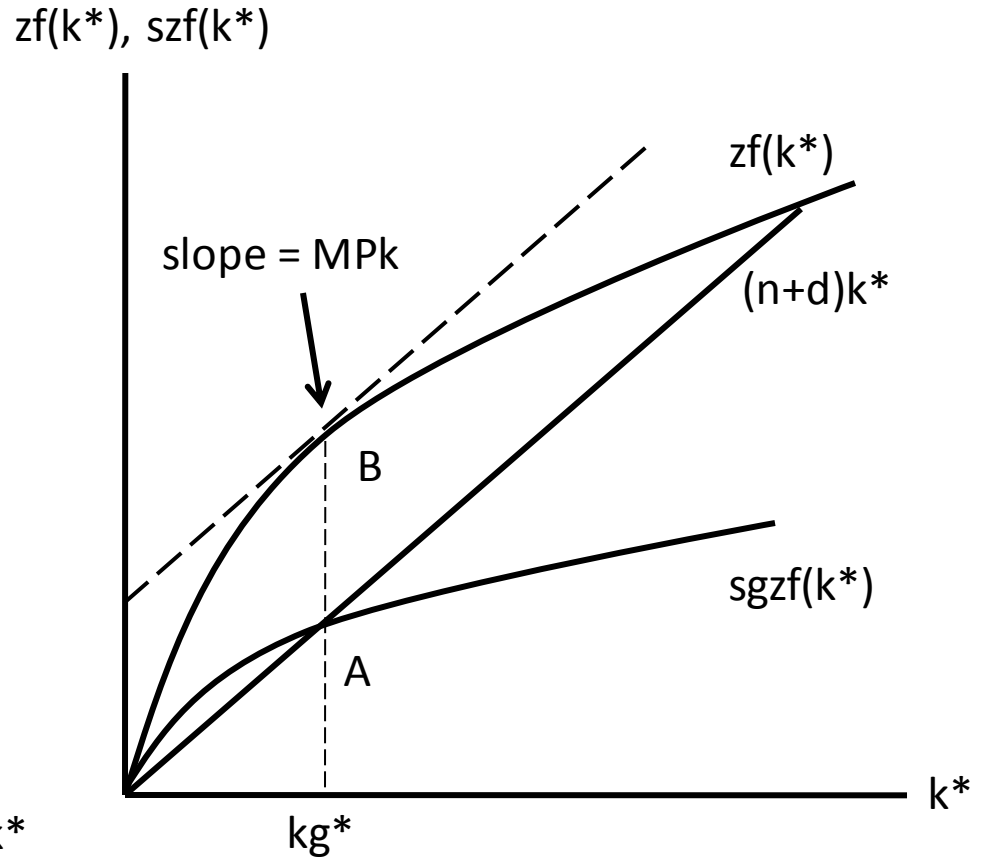
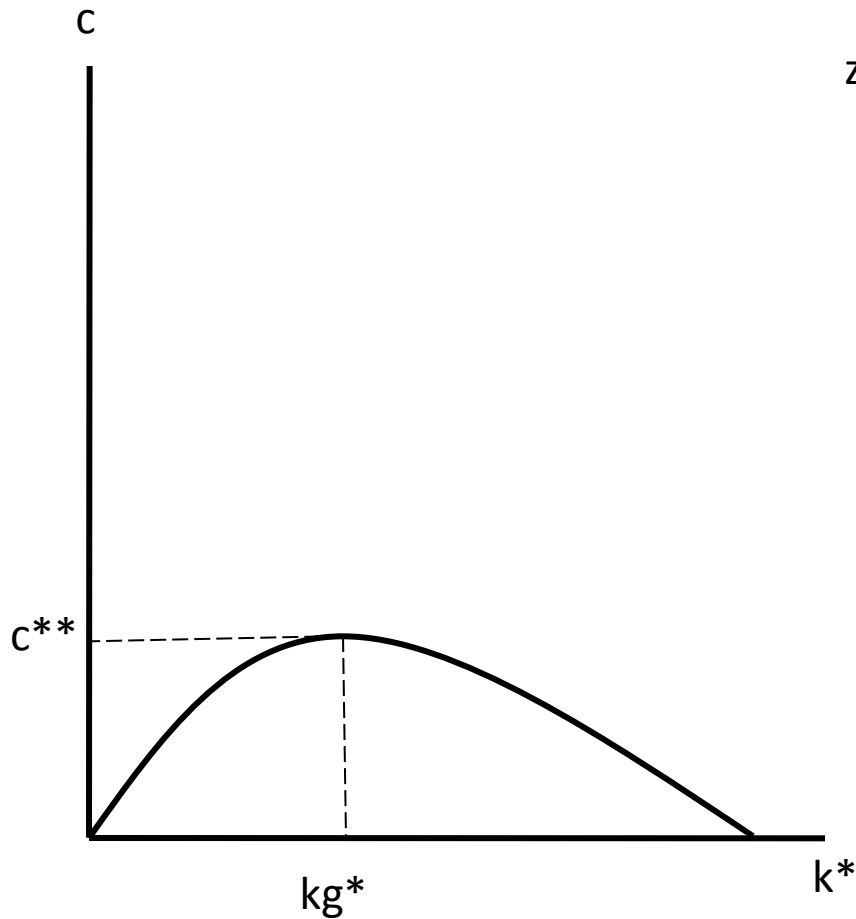
$$MP_k = n + d$$

# Golden-rule $s_g$

- The rate with max.  $c^{**}$  is the 'golden-rule' savings rate ( $s_g$ ).



# Maximized $c^{**}$ and golden-rule $s_g$



# Golden-rule $s_g$ and policy?

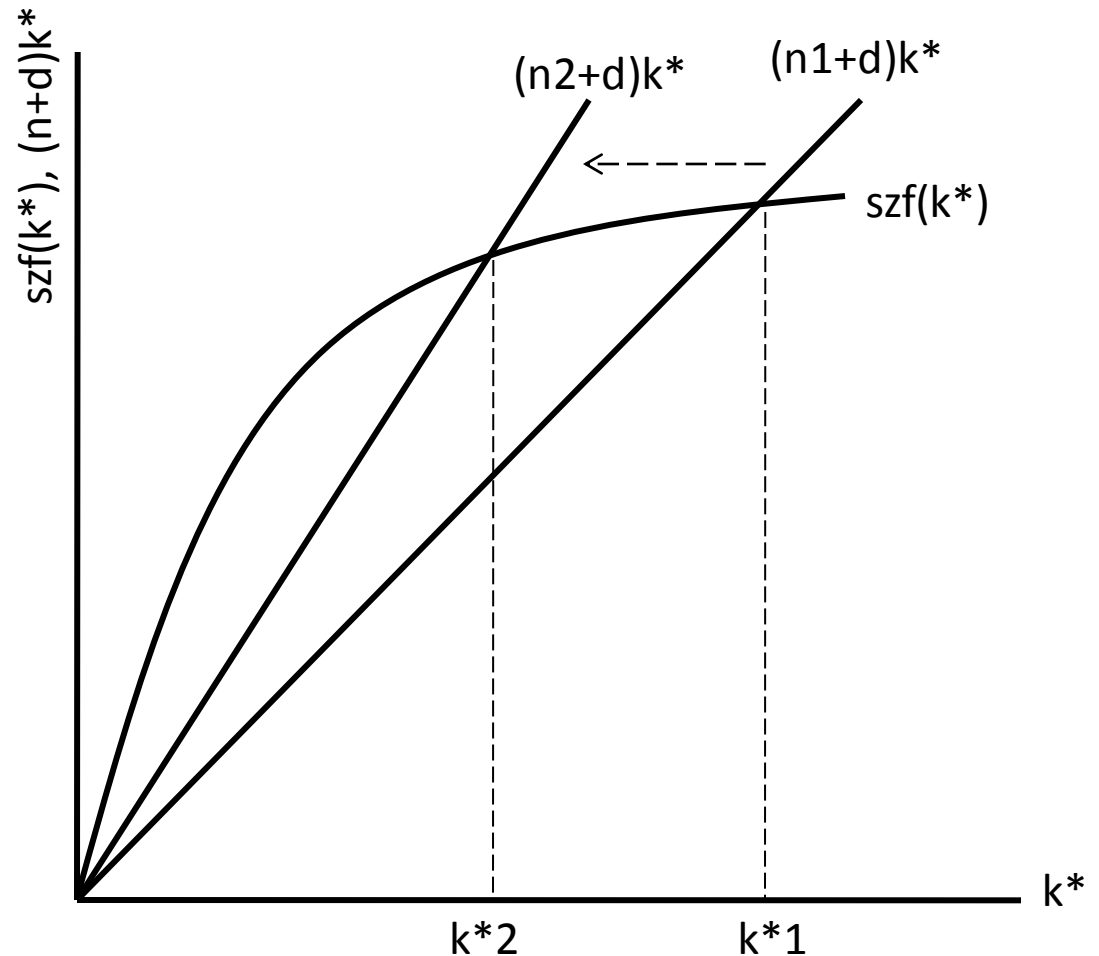
- With the golden-rule  $s_g$ ,  $c^{**}$  is maximized and steady in all periods.
  - Other  $s$  results in different  $c^*$  which is not max.
- Should we achieve  $s_g$  if the current  $s \neq s_g$ ?
  - A sacrifice of current  $c$  to build up a larger capital stock in the future is needed; is it worth?
  - $s$  depends on individuals' preference and the market for investment.

# Effect of an increase in $n$

- The increase in population growth ( $n_1$  to  $n_2$ ) rotates  $(n+d)k^*$  upwards.
- Decreased steady-state capital ( $k^*$ ) and output per worker ( $y^*$ ).
  - More workers ( $N^*$ ) produce larger output ( $Y^*$ ).
  - But falling productivity of labor results in lower output per worker ( $y^*$ ).
- The steady-state growth rate is higher at  $n_2$  for the capital stock ( $K$ ) and total output ( $Y$ ).

# A higher $n$ with lower $k^*$

- Higher population growth ( $n$ ) results in lower  $k^*$  and  $y^*$ .

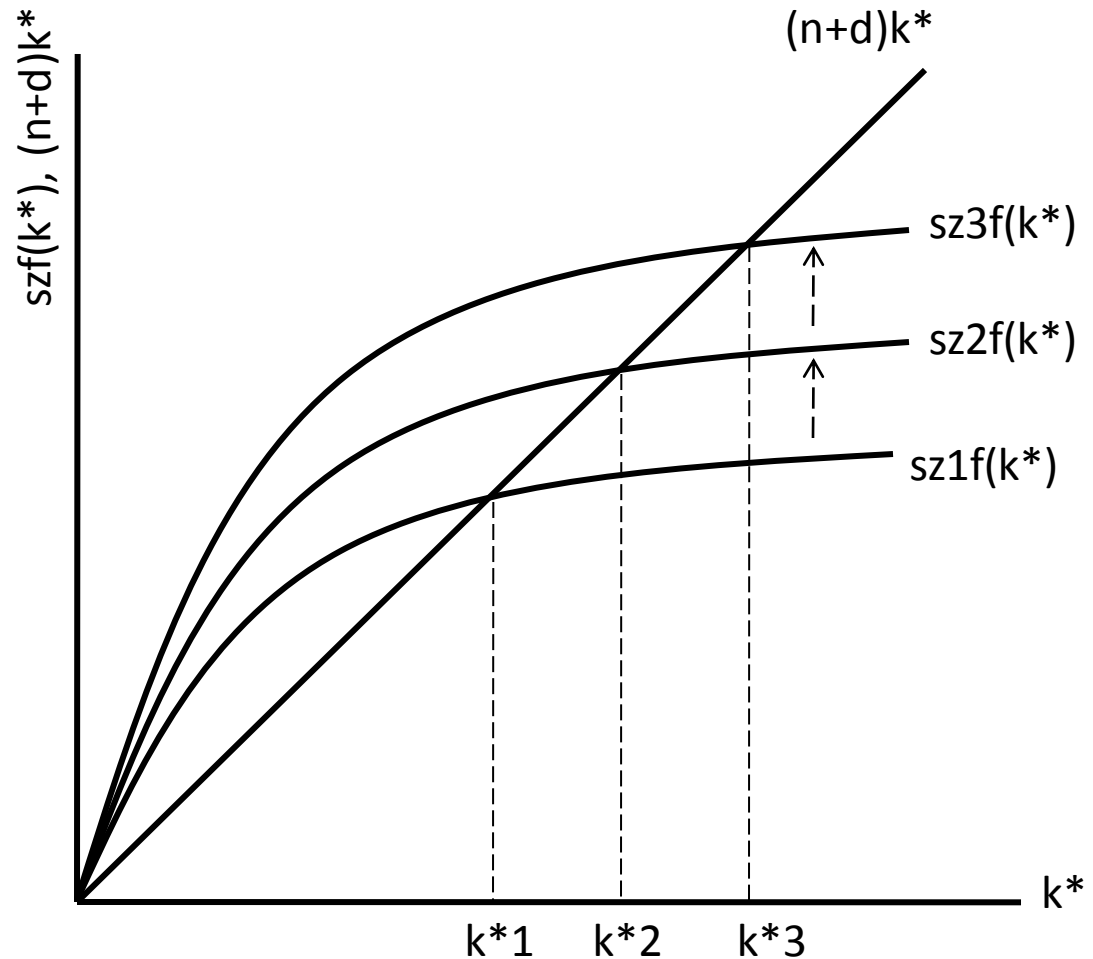


# Effect of an increase in $z$

- A rising  $s$  or falling  $n$  raises steady-state output per worker (living standards).
  - But the improvement will cease at some point ( $s$  cannot exceed 1;  $n$  cannot fall indefinitely).
- An increase in total factor productivity ( $z$ ) raises steady-state capital ( $k^*$ ) and output per worker ( $y^*$ ).
  - Sustained increases in  $z$  cause sustained increases in output per worker ( $y$ ).

# Sustained increases in $z$

- Sustained increases in  $z$  cause sustained improvement in  $y^*$ .



# Sources of sustained growth

- Growth from increases in **productive inputs**:
  - Physical capital accumulation,  $F(K, N)$ .
  - Human capital accumulation,  $F(K, H)$ .
- Growth from **total factor productivity** ( $z$ ):
  - Technical progress, inventions, better management and organization.
  - Weather, improved government regulations, falling input prices.

# Solow model predictions

- In the long run, higher savings rate results in higher income per worker.
  - **Fact:** positive correlation between GDP per capita and the ratio of investment to GDP.
- An increase in population growth causes a decrease in income per worker.
  - **Fact:** negative correlation between population growth and GDP per capita

# Solow model predictions

- Under the similar economic structure, growth convergence will occur.
  - Starting of with lower “K” would require higher growth rate than those starting with higher “K”
  - Steady state is pinned down; low “K” will be growing faster
    - Low “K” = higher MPK, marginal increase in output is more → higher growth, especially when the base is small.

# Engine of growth in Thailand

- Using Solow model, we can back out the contribution of each factor on long-term growth
- Let assume that Thailand's production function given by  $Y = zK^\alpha L^{1-\alpha}$ 
  - The function is CRTS.
  - $\alpha$  = Share of capital income to GDP = 1/3 (approx.)

# Engine of growth in Thailand

- We know that

$$\% \Delta Y = \% \Delta z + \alpha \% \Delta K + (1 - \alpha) \% \Delta L$$

- Given the capital growth, labor growth, and GDP growth, we can back out the  $\% \Delta z$ 
  - This term is called the Solow residual
  - This term is commonly used to capture the “total factor productivity”

# Engine of growth in Thailand

	1999-2006	2007-2009	2010 - 2016
K	2.596	2.512	2.54
L	1.508	1.348	0.516
Z	2.484	1.132	1.372
Y	4.7	3.236	3.204

- A decline in long-term growth! Why?
  - Capital investment averagely grows at a stable rate over the past 20 years!
    - Compared to period before that, the rate is lower
  - Demographic issue has suppressed growth process
    - Lower labor force growth!
  - After the GFC, Thailand has experienced a decline in TFP growth!