Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

1 a) find $\hat{\beta}_1$ and $\hat{\beta}_2$ with OLS and Regression model meaning

$$\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{\Sigma(X_i-\bar{X})(Y_i-\bar{Y})}{\Sigma(X_i-\bar{X})^2} = \frac{-174.2}{1098.8} = -0.1574 \ \#$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X} = 21.03 - (12.20)(-0.1574) = 22.9503 \ \#$$

* The regression model shows relationship between 2 variables.
* In this question we use the given information to find the estimators ($\hat{\beta}_1$ and $\hat{\beta}_2$) and create the sample regression function.

b.) $$r^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\Sigma \hat{u}_i^2}{\Sigma(Y_i-\bar{Y})^2} = 1 - \frac{873.14}{882.97} = 0.0111 \ \#$$

* r^2 describes the measurement of 'goodness of fit' of the fitted regression line comparing to the estimator $\bar{Y}$
* $r^2 > 0.995$ means the calculation of regression equation is quiet accuracy, however, its not true all the time depends on the information and the person who calculate.
* in this case, generally, r^2 is very low so it can be conclude that the calculation is not accuracy or the line does not fit well.

c.) $$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 x_i = 22.9503 + (-0.1574)(5) = 22.1633 \ \#$$

* $\hat{Y}_i$ is the estimator of $E(Y|X_i)$
* since we use the estimator to find $\hat{Y}_i$; there will no error in the function ($\hat{u}_i$)

d.) $$var(u_i) = \sigma^2 = \frac{\Sigma u_i^2}{n-k} = \frac{\Sigma(Y_i-\hat{Y})^2}{n-k} = \frac{873.14}{30-2} = 31.1836 \ \#$$

$$var(\hat{\beta}_1) = \frac{\Sigma X_i^2}{n\Sigma x_i^2}\sigma^2 = \frac{5564}{30(1{,}098.8)} \times \frac{873.14}{28} = 5.2635 \ \#$$

$$var(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma x_i^2} = \frac{873.14}{28 \times 1098.8} = 0.0284 \ \#$$

e.) coefficient different from zero at 0.05 ; $0 < \alpha < 1$

Step 1 $H_0 : \beta_2 = 0$ - Null hypothesis

$H_a : \beta_2 \neq 0$ - Alternative hypothesis

Step 2 $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\hat{\sigma}_{\hat{\beta}_2}} = \dfrac{-0.1574 - 0}{\sqrt{0.0284}} = -0.9340$ #

Step 3 $\alpha = 0.05$ $\beta_2 = 0$

The lower bound : $t_{\frac{\alpha}{2}} = t_{0.025}$; d.f = n-k = 30-2 = 28

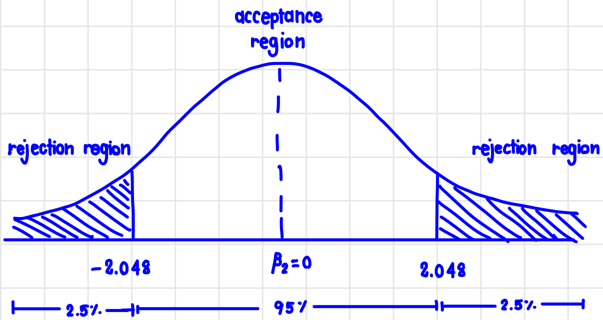
$= -2.048$ #

The upper bound : $t_{\frac{\alpha}{2}} = t_{0.025}$; d.f = n-k = 30-2 = 28

$= 2.048$ #

* note : using t cal

Step 4



* T cal = -0.9340 lies on the acceptance region, we cannot reject the null hypothesis at the significant level of 95%.

* In other words, we cannot say for sure that β_2 is not zero 95 out of 100 times when we sample.

f.) coefficient less than zero at 0.01 ; $0 < \alpha < 1$

Step 1 $H_0 : \beta_2 \geqslant 0$ - Null hypothesis

$H_a : \beta_2 < 0$ - Alternative hypothesis

Step 2 $t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \dfrac{-0.1574 - 0}{\sqrt{0.0284}} = -0.9340$ #

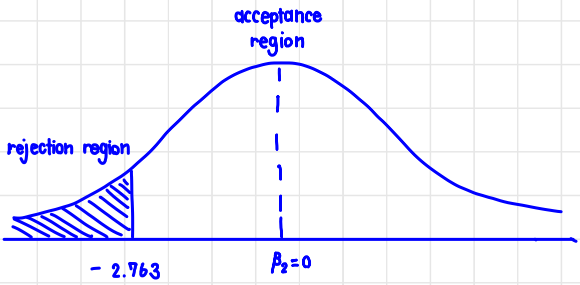
Step3 $\alpha = 0.01$ $\beta_2 = 0$

The lower bound : $t_{\frac{\alpha}{2}} = t_{0.005}$; d.f = n-k = 30-2 = 28

$= -2.763$ #

* note : using t cal

Step 4



* We use an one tail test since we want to make sure that β_2 is less than zero or not.

* T cal = -0.9340 lies on the acceptance region, we cannot reject the null hypothesis at the significant level of 99%.

* In other words, we cannot say for sure that β_2 is less than zero 99 out of 100 times when we sample.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = \overset{\hat{\beta}_1}{7{,}836} - \overset{\hat{\beta}_2}{502.4}X_i$$
$$(52) \qquad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11, n = 11

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

a) $\hat{\beta}_2 = -502.4$ #

* The β_2 in function makes sense as it is a negative number when the car is getting more older the price will fall.

b) $x_0 = 5$ $\alpha = 0.05$ $\bar{x} = 7.45$ $\hat{\sigma}^2 = 212,877$ $\Sigma(x_i - \bar{x})^2 = 78.73$ $n = 11$

$$\hat{Y}_0 = 7836 - 502.4x_0$$

$$= 7836 - 502.4(5) = 5,324 \text{ \#}$$

Step 1 $$var(\hat{Y}_0) = \sigma^2\left(\frac{1}{n} + \frac{(x_0-\bar{x})^2}{\Sigma(x_0-\bar{x})^2}\right)$$

$$= 212,877\left(\frac{1}{11} + \frac{(5-7.45)^2}{78.73}\right)$$

$$= 35,582.5345 \text{ \#}$$

Step 2 $$\sigma\hat{Y}_0 = \sqrt{35,582.5345} = 188.6333 \text{ \#}$$

Step 3 Find the 95% of CI for $E(Y|X_0 = 5)$

Upper bound : $\hat{Y}_0 + (t_{\frac{\alpha}{2}}\sigma\hat{Y}_0) = 5,324 + (t_{0.025}(188.6333))$

$= 5710.3209$ #

Lower bound : $\hat{Y}_0 - (t_{\frac{\alpha}{2}}\sigma\hat{Y}_0) = 5,324 - (t_{0.025}(188.6333))$

$= 4937.6790$ #

Step 4 $$4937.6790 \leq \hat{Y}_0 \leq 5710.3209 \text{ \#}$$

* The market price when the car is 5 years is averagely range in CI as between 4937.6790 and 5710.3209 USD.

c) multiply x by 10 ; new SRF and standard error

$$\hat{Y}_0 = 7836 - 502.4(10)\,X_0$$

$$= 7836 - 5024\,X_0$$

$$\hat{\beta}_2 = -5{,}024 \quad \#$$

$$\text{var}(\beta_2) = \frac{\sigma^2}{\Sigma x_i^2} = \frac{212{,}877}{78.73} = 2{,}703.8867 \quad \#$$

$$\text{se.}(\beta_2) = 411.8 \times 10 = 4{,}118 \quad \#$$

d) $X_0 = 10$

$$\hat{Y}_0 = 7836 - 502.4\,X_0$$

$$= 7836 - 502.4(10)$$

$$= 2{,}812 \quad \#$$

Elasticity . $\underbrace{\frac{dY}{dX}}_{\beta_2} \cdot \frac{X}{Y} = (-502.4)\,\frac{10}{2{,}812} = -1.7866 \quad \#$