Quiz 1: Date: April 19, 2022 from 11.00-12.30

Question 1 (10 Points) Score....................

Consider the one-period model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$U(C) = ln(C)$$

Also, let $\frac{C_1}{C_0}$ is distributed as log-normal with mean equals μ_c and its variance is σ_c.
Please read and answer the following questions carefully and completely.

Score....................

Question 1.1 (10 marks) Calculate the risk free rate R_f in terms of the individual's consumption, C_0 and C_1. Then,explain the relationship between the level of consumption and the risk free rate in this economy.

From the optimal intertemporal decision

$$u'(C_0) = R_f \delta E[u'(C_1)]$$

$$u'(C_1) = \frac{1}{C_1}$$

$$\frac{1}{R_f} = \frac{\delta E[u'(C_1)]}{u'(C_0)}$$

$$u'(C_0) = \frac{1}{C_0}$$

$$\frac{1}{R_f} = \delta E\left[\frac{C_0}{C_1}\right]$$

Ans: this equation implies that if the risk free rate is high, the expected growth in consumption will also be high.

Score.....................

Question 1.2 (10 marks) Calculate the elasticity of intertemporal substitution in this setting. If in the next year, the interest rate is falling, Will the individual's consumption level increase or decrease? Why? To support your answer, use the concepts of income effect and substitution effect.

If there is only a risk-free asset and nonrandom labor income, so that C_1 is nonstochastic, equation from 1.1 is

$$R_f = \frac{1}{\delta}\frac{C_1}{C_0}$$

$$\frac{\partial R_f}{\partial \frac{C_1}{C_0}} = \frac{1}{\delta}$$

$$= \frac{R_f}{C_1} \cdot C_0$$

$$\epsilon = \frac{\partial \frac{C_1}{C_0}}{\partial R_f} \cdot \frac{R_f}{\frac{C_1}{C_0}} = \left[\frac{C_1}{R_f C_0}\right] R_f \cdot \frac{C_0}{C_1}$$

$$\epsilon = 1$$

Ans. the elasticity is equal to 1, this mean that an increase in interest rate will raise the consumption in the next in a one-to-one relation.

As the substitution effect would increase the consumption next period and income effect would raise the consumption in this period.

For this case, it can be implied that the substitution effect and income effect are completely offset each other.

Score....................

Question 1.3 (10 marks) Solve for the pricing kernel P_i of any risky asset i in this economy. Then explain the meaning of this pricing kernel.

The pricing kernel can be derived by

$$m_{01} = \frac{\delta u'(C_1)}{u'(C_0)} \qquad u'(C_1) = \frac{1}{C_1} \qquad u'(C_0) = \frac{1}{C_0}$$

$$= \delta \frac{\frac{1}{C_1}}{\frac{1}{C_0}}$$

$$m_{01} = \delta \frac{C_0}{C_1}$$

From $P_i = E\left[\delta \frac{u'(C_1)}{u'(C_0)} x_i\right]$

this implies that any asset that pay the payoff in the state where C_1 is high, it will cause the marginal utility ($u'(C_1)$) to be low. Therefore, those types of asset will not be highly value, and vice versa.

Score.....................

Question 1.4 (10 marks) Calculate Hansen-Jaganathan Bound and explain the meaning.

HJ bound states that

$$\left|\frac{E(R_i) - R_f}{\sigma_{R_i}}\right| \leq \frac{\sigma_{m_{01}}}{E(m_{01})}$$

$$m_{01} = \frac{\delta c_0}{c_1} = \delta\left(\frac{c_1}{c_0}\right)^{-1} = \delta e^{-\ln\frac{c_1}{c_0}}$$

$$\frac{\sigma_{m_{01}}}{E(m_{01})} = \frac{\sqrt{Var(\delta e^{-\ln\frac{c_1}{c_0}})}}{E(\delta e^{-\ln\frac{c_1}{c_0}})}$$

$$= \frac{\sqrt{E(e^{-2\ln\frac{c_1}{c_0}}) - E(e^{-\ln\frac{c_1}{c_0}})^2}}{E(e^{-\ln\frac{c_1}{c_0}})}$$

$$= \sqrt{\frac{E(e^{-2\ln\frac{c_1}{c_0}})}{E(e^{-\ln\frac{c_1}{c_0}})^2} - 1}$$

$$= \sqrt{\frac{e^{-2\mu_c + 2\sigma_c}}{e^{-2\mu_c + \sigma_c}} - 1}$$

$$= \sqrt{e^{\sigma_c} - 1}$$

$$\approx \sqrt{1 + \sigma_c - 1}$$

$$\approx \sqrt{\sigma_c}$$

This means that the absolute value of sharpe ratio for all assets will be bound by the value of $\sqrt{\sigma_c}$ from the equation

$$\left|\frac{E(R_i) - R_f}{\sigma_{R_i}}\right| \leq \sqrt{\sigma_c}$$