1. Given this information

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$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n}(X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_iY_i = 7{,}524$	$\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n}(Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

$$\hat{\beta}_1 = \overline{Y} - \hat{\beta}_2\overline{X} = 21.03 - (-0.1585)(12.20) = 22.9637$$

$$\hat{\beta}_2 = \frac{\sum y_i x_i}{\sum x_i^2} = \frac{-174.20}{1098.8} = -0.1585$$

- Meaning of $\hat{\beta}_2$, when X increases by 1 on average Y will decrease by 0.1585
- meaning of $\hat{\beta}_1$, when X is zero Y equals to 22.9637

b) Find r^2 and explain its meaning.

$$r^2 = 1 - \frac{\sum \hat{u}_i^2}{\sum y_i^2} = 1 - \frac{873.14}{882.97} = 0.0111$$

This means that about 1.11 % of variations in Y can be explained by variations in X.

c) If $X_i = 5$, estimate the value of Y_i and explain its meaning.

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i \quad , X_i = 5$$

$$\hat{Y}_i = 22.9637 + (-0.1585)(5) = 22.1712$$

This means that when $X_i = 5$, the corresponding conditional mean value of Y is 22.1712

d) Find the estimators of $\operatorname{var}(u_i)$, $\operatorname{var}(\hat{\beta}_1)$ and $\operatorname{var}(\hat{\beta}_2)$

$$\operatorname{var}(u_i) = \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-k} = \frac{873.14}{30-2} = 31.1835$$

$$\operatorname{var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum x_i^2} = \frac{31.1835}{1098.8} = 0.0283$$

$$\operatorname{var}(\hat{\beta}_1) = \frac{\sum X_i^2}{n\sum x_i^2} \cdot \hat{\sigma}^2 = \frac{5564}{(30)(1098.8)} \cdot 31.1835 = 5.2634$$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

Step 1 : state the hypothesis

$H_0 : \beta_1 = 0$	$H_0 : \beta_2 = 0$
$H_a : \beta_1 \neq 0$	$H_a : \beta_2 \neq 0$

Step 2 : calculate test statistics

$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}_{\hat{\beta}_1}} = \frac{22.9637 - 0}{\sqrt{5.2634}} = 10.0094$$

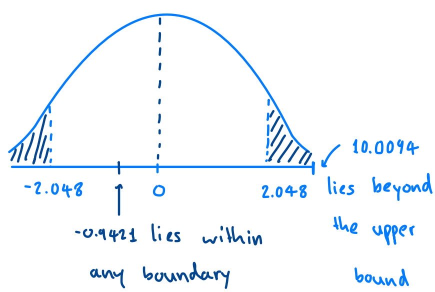
$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\hat{\sigma}_{\hat{\beta}_2}} = \frac{-0.1585 - 0}{\sqrt{0.0283}} = -0.9421$$

Step 3 : state the decision rule

$\alpha = 0.05$, $n-k = 30-2 = 28$

The lower bound $= -t_{\frac{\alpha}{2}} = -t_{0.025,28} = -2.048$

The upper bound $= t_{\frac{\alpha}{2}} = t_{0.025,28} = 2.048$



Step 4 : conclude the test

$t_{cal} = 10.0094 >$ upper bound
we can reject the null hypothesis, at the significant level of 95 %

$t_{cal} = -0.9421$, which is in the acceptance region
we cannot reject the null hypothesis, at the significant level of 95 %

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

Step 1 : state the hypothesis

$H_0 : \beta_1 \geq 0$ | $H_0 : \beta_2 \geq 0$

$H_a : \beta_1 < 1$ | $H_a : \beta_2 < 0$

Step 2 : calculate test statistics

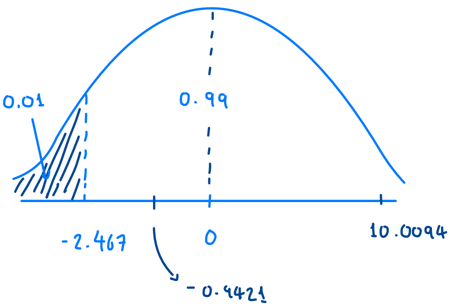
$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\delta \hat{\beta}_1} = \frac{22.9637 - 0}{\sqrt{5.2634}} = 10.0094$$

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\delta \hat{\beta}_2} = \frac{-0.1585 - 0}{\sqrt{0.0283}} = -0.9421$$

Step 3 : state the decision rule

$\alpha = 0.01$, $n - k = 30 - 2 = 28$

Lower bound => $-t_{\alpha} = -2.467$



Step 4 : conclude the test

t_{cal} > lower bound, we fail to reject H_0 at 0.01 level of significance.

β_1 and β_1 are not less than zero.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = \underbrace{7{,}836}_{\hat{\beta}_1} - \underbrace{502.4}_{\hat{\beta}_2}X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

Yes, it does because as the sign of $\hat{\beta}_2$ is negative it means that the older of the car, the cheaper of it. Since the buyer expects the less use of the car and they would pay less for the older car.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

Predict $\hat{Y}$ at $X = 5$, find 95% CI for $E(Y_0|X_0) = 5$

$$\hat{Y} = \hat{\beta}_1 - \hat{\beta}_2 X_i$$

$$\hat{Y} = 7836 - 502.4(5) = 5324$$

$$var(\hat{Y}_0) = \hat{\sigma}^2\left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right] = 212{,}877\left[\frac{1}{11} + \frac{(5 - 7.45)^2}{78.73}\right] = 35582.5345$$

$$P\left[\hat{Y}_0 - (t_{\frac{\alpha}{2}} \cdot \hat{\sigma}\hat{Y}_0) \leq E(Y_0|X_0 = 5) \leq \hat{Y}_0 + (t_{\frac{\alpha}{2}}\ \hat{\sigma}\hat{Y}_0)\right] = 1 - \alpha$$

$$P\left[5324 - (2.262 \times 35582.5345) \leq E(Y_0|X_0 = 5) \leq 5324 + (2.262 \times 35582.5345)\right] = 0.95$$

$$P\left[-75163.6930 \leq E(Y_0|X_0 = 5) \leq 85811.6930\right] = 0.95$$

$\Rightarrow$ The 95% confidence interval for market price of a 5 years old car is within $(-75163.6930, 85811.6930)$.

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

Data scaling : The original SRF ; $\hat{Y}_i = 7836 - 502.4X_i$ $\quad Y = \text{USD},\ X = 1 \text{ year}$

$$\begin{matrix} & (52) & (411.8) \end{matrix}$$

If we multiply all the X with 10, we will get

$$\hat{Y}_i = 7836 - 50.24X_i \qquad Y = \text{USD},\ X = 0.1 \text{ year}$$

$$\begin{matrix} & (52) & (41.18) \end{matrix}$$

d) Calculate the elasticity of market price when a car is 10 years old.

$X = 10$; $\hat{Y}_i = \hat{\beta}_1 - \hat{\beta}_2 X_i \Rightarrow \hat{Y}_i = 7836 - 502.4(10)$

$$= 2812$$

$$\text{Elasticity} = \frac{dy}{dx}\left(\frac{x}{y}\right) = -\beta_2\left(\frac{x}{y}\right) = -502.4\left(\frac{10}{2812}\right) = -1.79$$

we can say that for 1 % increases of the car's age from 10 years, there will be 1.79 % decrease in its market price.