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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

   a. $\sum_{i=1}^{5}(a+bx_i)$ $= 5a + b\sum_{i=1}^{5} x_i$

   $= 5a + b(x_1 + x_2 + x_3 + x_4 + x_5)$

   b. $\sum_{y=0}^{5} f(x+y)$ $= f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$

   c. $\sum_{i=1}^{10} i^2$ $= \frac{n(n+1)(2n+1)}{6} = \frac{10(10+1)(2(10)+1)}{6} = 385$

   d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y)$ $= \sum_{x=1}^{2}\left[2x + 2 + 3\right] = \sum_{x=1}^{2} 2x + 5 = 2(1) + 2(2) + 5 = 11$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

   a. Find the value of b

   $f(x) = P(X = x) = 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1$

   $8b = 1$

   $b = 1/8$

   b. Find the answer for $P(X \le 2)$

   $P(X \le 2) = 1 - P(X > 2)$

   $= 1 - P(X = 3) - P(X = 4)$

   $= 1 - (0.5)(0.125) - (0.25)(0.125) = 0.90625$

   c. Find the answer for $P(-2 \le X \le 3)$

   $P(-2 \le X \le 3) = 1 - P(X = 4)$

   $= 1 - (0.25)(0.125)$

   $= 0.96875$

   d. Find the answer for $P(X \ge 1)$

   $P(X \ge 1) = 1 - P(X < 1)$

   $= 1 - P(X = 0) - P(X = -1) - P(X = -2)$

   $= 1 - (2.25)(0.125) - (1)(0.125) - (0.5)(0.125)$

   $= 0.53125$

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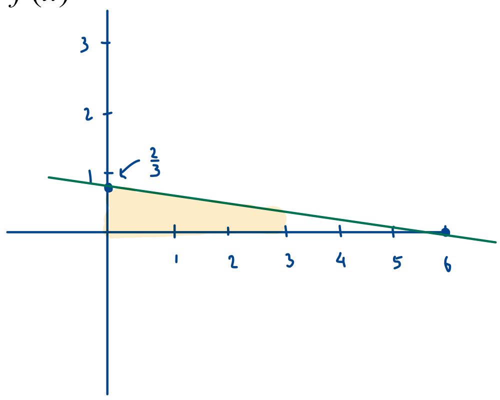
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$(x, y) = (6, \frac{2}{3})$

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \le X \le 3)$

$$P(1 \le X \le 3) = \int_1^3 f(x)\,dx$$

$$= -\frac{1}{18}x^2 + \frac{6}{9}x \Big|_1^3$$

$$= \left[-\frac{(3)^2}{18} + \frac{6(3)}{9}\right] - \left[-\frac{(1)^2}{18} + \frac{6(1)}{9}\right] = -\frac{9}{18} + \frac{18}{9} + \frac{1}{18} - \frac{6}{9} = -\frac{8}{18} + \frac{12}{9} = \frac{16}{18}$$

c. Find the answer for $P(X \ge 2)$

$$P(X \ge 2) = \int_2^3 f(x)\,dx$$

$$= -\frac{1}{18}x^2 + \frac{6}{9}x \Big|_2^3$$

$$= \left[-\frac{(3)^2}{18} + \frac{6(3)}{9}\right] - \left[-\frac{(2)^2}{18} + \frac{6(2)}{9}\right] = -\frac{9}{18} + \frac{18}{9} + \frac{4}{18} - \frac{12}{9} = \frac{7}{18}$$

d. Find the expected value of X

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

$$= \int_0^3 x\left(-\frac{1}{9}x + \frac{6}{9}\right)dx$$

$$= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x\,dx$$

$$= \frac{-x^3}{27} + \frac{6x^2}{18} \Big|_0^3$$

$$= -\frac{(3)^3}{27} + \frac{6(3)^2}{18}$$

$$= -\frac{27}{27} + \frac{\overset{3}{\cancel{6}} \cdot 9}{\underset{2}{\cancel{18}}}$$

$$= -1 + 3$$

$$= 2$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

X/Y	1	2	3	4	5	6	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

   b. Find the marginal probability distribution function (PDF) of X

the marginal probability of x $P(X=x)$ is $\frac{1}{6}$

   c. Find the marginal probability distribution function (PDF) of Y

the PDF, $P(Y=y)$ is $\frac{1}{2}$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1 $P(X|Y=1)$

x	1	2	3	4	5	6
$P(X=x \mid Y=1)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

   e. Find the expected value of X given Y is equal to 1

$$E(X|Y=1) = \Sigma x_i P(X=x_i|Y=1) = \frac{\Sigma x_i P(X=x_i, Y=1)}{P(Y=1)} = \frac{1}{P(Y=1)} \Sigma x_i P(X=x_i, Y=1)$$

$$= \frac{1}{0.5}\left[(1\cdot\tfrac{1}{12}) + (2\cdot\tfrac{1}{12}) + (3\cdot\tfrac{1}{12}) + (4\cdot\tfrac{1}{12}) + (5\cdot\tfrac{1}{12}) + (6\cdot\tfrac{1}{12})\right] = \frac{7}{2}$$

   f. Find the variance of X given Y is equal to 1

$$Var(X|Y=1) = \Sigma (X - E(X|Y=1))^2 \cdot P(X|Y=1)$$

$$= \left[\left((1-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right) + \left((2-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right) + \left((3-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right) + \left((4-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right)\right.$$

$$\left. + \left((5-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right) + \left((6-\tfrac{7}{2})^2\cdot\tfrac{1}{6}\right)\right]$$

$$= \frac{10}{3}$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{X}) = E\left(\frac{1}{N}\sum_{i=1}^{3} X_i\right)$$

$$= \frac{1}{N} E(X_1 + X_2 + X_3)$$

$$= \frac{1}{N}\left[E(x_1) + E(x_2) + E(x_3)\right]$$

$$= \frac{1}{3}\left[\mu_x + \mu_x + \mu_x\right]$$

$$= \frac{1}{\cancel{3}} \cdot \cancel{3}\mu_x = \mu_x$$

$$Var(\bar{x}) = Var\left(\frac{1}{N}\sum_{i=1}^{3} X_i\right)$$

$$= \frac{1}{N^2} Var(X_1 + X_2 + X_3)$$

$$= \frac{1}{N^2}\Big[Var(x_1) + Var(x_2) + Var(x_3) + 2Cov(x_1, x_2) + 2Cov(x_1, x_3) + 2Cov(x_2, x_3)\Big]$$

$$= \frac{1}{3^2}\left[\frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_x^2\right]$$

$$= \frac{1}{3^2} \cdot \frac{9}{4}\sigma_x^2 = \frac{\sigma_x^2}{4}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

   a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{X}) = E\left(\frac{1}{N}\sum_{i=1}^{4} X_i\right)$$

$$= \frac{1}{N} E(X_1 + X_2 + X_3 + X_4)$$

$$= \frac{1}{N}\left[E(x_1) + E(x_2) + E(x_3) + E(x_4)\right]$$

$$= \frac{1}{4}\left[\mu_x + \mu_x + \mu_x + \mu_x\right]$$

$$= \frac{1}{4} \cdot 4\mu_x = \mu_x$$

$$Var(\bar{x}) = Var\left(\frac{1}{N}\sum_{i=1}^{4} X_i\right)$$

$$= \frac{1}{N^2} Var(X_1 + X_2 + X_3 + X_4)$$

$$= \frac{1}{N^2}\left[Var(x_1) + Var(x_2) + Var(x_3) + Var(x_4)\right]$$

$$= \frac{1}{4^2}\left[\sigma_x^2 + \sigma_x^2 + \sigma_x^2 + \sigma_x^2\right]$$

$$= \frac{1}{16} \cdot 4\sigma_x^2 = \frac{\sigma_x^2}{4} = 0.25\sigma_x^2$$

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b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$\tilde{X} = \frac{1}{4}(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$

$E(\tilde{X}) = E(\frac{1}{N}\sum_{i=1}^{4} X_i)$

$= \frac{1}{4} E(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$

$= \frac{1}{4}\left[E(0.5X_1) + E(X_2) + E(0.5X_3) + E(2X_4)\right]$

$= \frac{1}{4}\left[0.5E(X_1) + E(X_2) + 0.5E(X_3) + 2E(X_4)\right]$

$= \frac{1}{4}(0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x)$

$= \frac{1}{4} \cdot 4\mu_x = \mu_x$

$\tilde{X}$ is unbiased estimator of μ

$Var(\tilde{X}) = Var(\frac{1}{N}\sum_{i=1}^{4} X_i)$

$= \frac{1}{4^2} Var(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$

$= \frac{1}{4^2}\left[Var(0.5X_1) + Var(X_2) + Var(0.5X_3) + Var(2X_4)\right]$

$= \frac{1}{4^2}\left[0.25Var(X_1) + Var(X_2) + 0.25Var(X_3) + 4Var(X_4)\right]$

$= \frac{1}{4^2}\left[0.25\sigma_x^2 + \sigma_x^2 + 0.25\sigma_x^2 + 4\sigma_x^2\right]$

$= \frac{5.5\sigma_x^2}{16}$

$= 0.34\sigma_x^2$

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$\bar{X}$ is a more efficient estimator of μ_x than $\tilde{X}$ because it has smaller variance.