Quiz 2: Date: May 5, 2022 from 11.00-12.30

Question 1 (40 marks) Score.....................

Consider the Muliperiod model of consumption and portfolio choice. Let an individual in this economy has the utility function as follow:

$$\max_{C_s,\omega_s,\forall_t} E_t\left[\sum_{s=t}^{T-1}\delta^s\left(\frac{C_s^{(1-\gamma)}}{1-\gamma}\right)+\delta^T\left(\frac{W_T^{(1-\gamma)}}{1-\gamma}\right)\right]$$

Assume that there is no wage income $(y_t = 0\,\forall t)$ and a constant risk-free rate return asset , $R_{ft} = R_f$. Also asssume that n=1 and the return of a single risky asset, R_{rt},is independently and identically distributed over time. Denote the proportion of wealth invested in the risky asset at date t as ω_t.

Please read and answer the following questions carefully and completely.

Score.....................

Question 1.1 (10 marks) Derive the first-order condition for the optimal consumption level and portfolio weight at date T-1, C^*_{T-1} and ω^*_{T-1}., and give an explicit expression for C^*_{T-1}

$$\max_{C_s,\omega_s,\forall_t} E_t\left[\sum_{s=t}^{T-1}\delta^s\left(\frac{C_s^{(1-\gamma)}}{1-\gamma}\right)+\delta^T\left(\frac{W_T^{(1-\gamma)}}{1-\gamma}\right)\right]$$

F.O.C.

$$= \delta^{T-1}\ln(C^*_{T-1}) + \delta^T E_{T-1}\left[\ln\left(R^*_{T-1}\left(W_{T-1} - C^*_{T-1}\right)\right)\right]$$

$$= \delta^{T-1}\left[(1+\delta)\ln(W_{T-1}) + H_{T-1}\right]$$

$$C^*_{T-1} = \frac{1-\gamma}{1+\delta} W_{T-1}$$

Score.....................

Question 1.2 (10 marks) Solve for the form of J(W_{T-1},T-1).

$$\max_{C_s,\omega_s,\forall_t} E_t\left[\sum_{s=t}^{T-1}\delta^s\left(\frac{C_s^{(1-\gamma)}}{1-\gamma}\right)+\delta^T\left(\frac{W_T^{(1-\gamma)}}{1-\gamma}\right)\right]$$

$$J(W_{T-1}, T-1) = \max_{C_{T-1},\{w_{i,T-1}\}} E_{T-1}\left[u(C^{(1-\gamma)}_{T-1,T-1}) + \beta(W^{(1-\gamma)}_{T,T})\right]$$

$$= \max_{C_{T-1},\{w_{i,T-1}\}} u(C^{(1-\gamma)}_{T-1,T-1}) + E_{T-1}\left[\beta(W^{(1-\gamma)}_{T,T})\right]$$

$$J(W_{T-1}, T-1) = \max_{C_{T-1},\{w_{i,T-1}\}} u(C^{(1-\gamma)}_{T-1,T-1}) + E_{T-1}\left[\beta(S_{T-1}R_{T-1,T})\right]$$

$$u(C^{(1-\gamma)}_{T-1,T-1}) - E_{T-1}\left[\beta_W(W^{(1-\gamma)}_{T,T})R_{T-1}\right] = 0$$

$$E_{T-1}\left[\beta_W(W^{(1-\gamma)}_{T,T})(R_{i,T-1} - R_{f,T-1})\right] = 0$$

$$u(C^{(1-\gamma)}_{T-1,T-1})$$

$$= E_{T-1}\left[\beta_W(W^{(1-\gamma)}_{T,T})\left(R_{f,T-1} + \sum_{i=1}^{N} w_{i,T-1}(R_{i,T-1} - R_{f,T-1})\right)\right]$$

$$= R_{f,T-1}E_{T-1}\left[\beta_W(W^{(1-\gamma)}_{T,T})\right]$$

$$J_W = (1-\gamma)E_{T-1}\left[\beta W_T R_{T-1}\right] \quad //$$

Score.....................

Question 1.3 (10 marks) Derive the first-order condition for the optimal consumption level and portfolio weight at date T-2, C^*_{T-2} and ω^*_{T-2}., and give an explicit expression for C^*_{T-2}

$$u_c(C^*_{T-2}, T-2) = E_{T-2}[J_W(W_{T-1}, T-1) R_{T-2}]$$

$$\frac{\delta^{T-2}}{C_{T-2}} = (1+\delta)\delta^{T-1} E_{T-2}\left[\frac{R_{T-2}}{S_{T-2} R_{T-2}}\right]$$

$$= \frac{(1+\delta)\delta^{T-1}}{W_{T-2} - C_{T-2}}(1-\gamma)$$

or $$C^*_{T-2} = \frac{1-\gamma}{1+\delta+\delta^2} W_{T-2}$$

Score.....................

Question 1.4 (10 marks) Solve for the form of J(W_{T-2},T-2).Based on the pattern for T-1 and T-2, provide expressions for the optimal consumption and portfolio weight at any date T-t, t=1,2,3,....

$$J(W_{T-2}, T-2)$$

$$= \max_{C_{T-2}, \{w_{i,T-2}\}} u(C_{T-2}^{(1-\gamma)}, T-2) + E_{T-2}[u(C_{T-1}^{(1-\gamma)}, T-1) + B(W_T^{(1-\gamma)}, T)]$$

depend on
future decisions
$C_{T-1}^{(1-\gamma)}$ and $\{w_{i,T-1}\}$