B.E. International Program
Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics
Semester 1/2017

Assignment 2 - Solution

Due 28th September 2017 (at the BE office, before 3 pm)

Note: We can't return your graded homework before the midterm exam. Please manage to photocopy your work before you turn in. You can check your work with the answer keys that will be posted after the lecture.

Question 1: *IS-LM model (Matrix Algebra)*

Consider the following IS-LM model:

Commodity market:

$$Y = C + I + G_0, \quad (G_0 > 0)$$

$$C = a + bY_d, \quad (0 < b < 1)$$

$$Y_d = Y - T,$$

$$T = T_0 + tY, \quad (T_0 > 0,\ 0 < t < 1)$$

$$I = I_0 - kr, \quad (I_0 > 0,\ k > 0)$$

Money market:

$$M_s = M_0$$

$M_D = mY - hr, \quad (m > 0,\ h > 0)$

a. (2 points) Write a matrix form of the IS-LM equations with Y and r as the endogenous variables.

Ans.

$$\begin{bmatrix} 1 - b(1-t) & k \\ m & -h \end{bmatrix} \begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} a - bT_0 + I_0 + G_0 \\ M_0 \end{bmatrix}$$

b. (2 points) State the condition for the existence of the equilibrium national income and interest rate.

Ans.

$$\begin{vmatrix} 1 - b(1-t) & k \\ m & -h \end{vmatrix} \neq 0$$

$\Rightarrow \quad -h[1 - b(1-t)] - km \neq 0,$

Or $\quad -h + hb - hbt - km \neq 0$

Or $\quad h[1 - b(1-t)] + km \neq 0$

c. (4 points) Solve for the equilibrium level of national income and interest rate by using Cramer's rule.

Ans.

$$Y^* = \frac{\begin{vmatrix} a-bT_0+I_0+G_0 & k \\ M_0 & -h \end{vmatrix}}{\begin{vmatrix} 1-b(1-t) & k \\ m & -h \end{vmatrix}} = \frac{h(a-bT_0+I_0+G_0)+kM_0}{h[1-b(1-t)]+km}$$

$$r^* = \frac{\begin{vmatrix} 1-b(1-t) & a-bT_0+I_0+G_0 \\ m & M_0 \end{vmatrix}}{\begin{vmatrix} 1-b(1-t) & k \\ m & -h \end{vmatrix}} = \frac{m(a-bT_0+I_0+G_0)-[1-b(1-t)]M_0}{h[1-b(1-t)]+km}$$

d. (2 points) Determine the rate of change of equilibrium national income with respect to the government expenditure (G), assuming that everything else remains constant.

Ans.

$$\frac{dY^*}{dG_0} = \frac{h}{h[1-b(1-t)]+km} > 0$$

Question 2: *Market equilibrium (Matrix Algebra)*

Given the following supply and demand functions:

$$Q_d = 100 - 3P$$

$$Q_s = 80 + 2P$$

a. (2 points) Write the equilibrium condition for this market, and translate the system of equations into matrix notation.

Eq'm condition: $Q_d = Q_s$.

System of equations:

$$\begin{aligned} Q_d - Q_s \quad &= 0 \\ Q_d \quad + 3P &= 100 \\ Q_s - 2P &= 80 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 80 \end{bmatrix}$$

b. (2 points) Use matrix inversion to solve for the equilibrium quantity and equilibrium price.

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow \rightarrow \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 100 \\ 80 \end{bmatrix} = \begin{bmatrix} 88 \\ 88 \\ 4 \end{bmatrix}$$

c. (6 points) Suppose that the government subsidizes the consumption of this good by giving the consumer \$5 per unit of the goods consumed. Write the new equilibrium condition for this market in the matrix form, and use Cramer's rule to solve for (i) the equilibrium price paid by the consumer, (ii) the price received by the producer, and (iii) the amount of money the government needs for this subsidization.

(i) $P_d^* = P_s^* - 5 = \$7 - \$5 = \$2$

(ii) $P_s^* = \$7$

(iii) $Q^* = 94; S^* = 94x\$5 = \$470$

Question 3: *Simple calculus*

Suppose that $Y = x^3 - 5x^2 + 7x - 5$

a. (2 points) Show the domain of X where the function exhibits the property of an increasing function.

$y' = 3x^2 - 10x + 7$

$3x^2 - 10x + 7 = (3x - 7)(x - 1)$

$y' > 0$ only when $x \in (-\infty, 1) \cup (7/3, \infty)$

b. (2 points) Define the domain set of X. Is the function concave for all over the domain?

$y'' = 6x - 10$

Concave when $y'' < 0$. That is when $x < \frac{10}{6}$.

c. (3 points) Find all the relative extreme points/values.

As in "a", two possible extreme points are 1 and 7/3. As you can see from "b", when x is less than 10/6, the function is concave. Since "1" is less than 10/6, the function is concave at x = 1. Thus, x=1 is the relative maximum point. Meanwhile, x = 7/3 is the relative minimum point because the function is convex at that value of x.

To find the maximum/minimum value, substitute x = 7/3 (y = -86/27) and x = 1 (y = -2) into the equation.

d. (3 points) Find global extreme points/values if domain of X is restricted to be in interval of (-7,10].

Check the two boundary points, x= -7 and x = 10. When x = 10, y = 565. When x = -7, y = -642. If one ranks all the four numbers, we will find that the largest number is 565. This means that x =10 is the global maximum point, with y = 565 as its associated global maximum value. *For the global minimum, there would be no answer for this case.* It would have been that X = -7 is the global maximum point if X = -7 were included in the interval of number under consideration. This is not the case because the problem states that you have to consider the value of X in an interval that does not include -7.

Question 4: Profit v.s. revenue function

Suppose that $C(Q) = \frac{1}{3}Q^3 - 7Q^2 + 111Q + 50$ and $Q = 100 - P$. Answer the following questions.

a. (2 points) Find the expression for the total revenue function in terms of Q.

   $R(Q) = PQ - C(Q) = (100 - Q)Q$

b. (2 points) Find the expression for the total profit function in terms of Q.

   $\pi(Q) = PQ - C(Q) = (100 - Q)Q - (\frac{1}{3}Q^3 - 7Q^2 + 111Q + 50)$

c. (4 points) What is the level of profit-maximizing output?

MR = 100 - 2Q

MC = $Q^2 - 14Q + 111$

Setting MR equal to MC, this yields us two values of Q, i.e Q = 11 and Q = 1.

When Q = 11, the profit function is concave.

So this Q is the level of profit-maximizing output.

d. (2 points) What is the level of maximized profit?

When Q = 11, profit is equal to 334/3.

Question 5: *Tax and revenue.*

Let the demand function be $P = 14 - 3Q$ and the supply function be $P = 4 + 2Q$. Suppose that the government imposes tax by \$t per unit of output. This tax is assumed to impose on consumer. Answer the following questions.

a. (1 points) Find the equilibrium under pre-tax situation. That is, when "t" is set to equal to zero.

   $14 - 3Q = 4 + 2Q$ ==> $Q = 2$ ==> $P = 8$.

b. (1 points) State the condition that links between consumer's and producer's price.

   $P^d = P^s + t$

c. (2 points) Find the equilibrium after tax. (Hint: your solution should be written in terms of "t".)

   Distinguish between the two prices by using different notation for price in each equation.

   That is, the model is written by using the following two equations.

   $P^d = 14 - 3Q^d$ and $P^s = 4 + 2Q^s$

   Plugging in the condition in "b" to the demand equation, and impose the equilibrium condition that $Q^d = Q^s$, we yield that

   $P^s + t = 14 - 3Q$ and $P^s = 4 + 2Q$

   Solve for Q, the answer is that $Q = \frac{10-t}{5}$.

   Given Q, the answer for producers' price is $P^s = \frac{40-2t}{5}$.

   In the equilibrium, consumers' price would be $P^d = P^s + t = \frac{40+3t}{5}$

d. (2 points) Calculate the consumers' and producers' burden. Which group is paying more for the tax under the equilibrium?

   Consumers are paying more. Their burden is $\frac{3}{5}$ of the total tax cost.

The remaining ⅖ are borne by producers.

e. (2 points) Find the expression of the revenue that the government can collect from the market under the equilibrium.
$TR = t * Q = t * (\frac{10-t}{5})$

f. (2 points) If the government were to collect tax so that total revenue is maximized, what is the appropriate level of unit tax, "t", that it should impose to the market?
$TR = t * Q = t * (\frac{10-t}{5}) = \frac{10t-t^2}{5}$ ==> $t^* = \$5.$ *Maximized revenue is* $25.

Question 6: *Production function.*

Suppose that a firm's short-run production function is given by

$$Q(L) = 6L^2 - L^3$$

where *Q(L)* is the output level, and *L* is the number of workers

a. (2 points) Derive the average product of labor (AP_L) and the marginal product of labor (MP_L).
$AP_L = 6L - L^2$; $MP_L = 12L - 3L^2$

b. (2 points) What size of the work force (L^{**}) maximizes the average output per labor, Q(L)/L?
Max $AP_L = 6L - L^2$

FOC: 6 - 2L = 0

⇒ $L^* = 3$

c. (2 points) Use calculus to show that the MP_L curve must cross the AP_L curve at its maximum point.

$$\frac{d(AP_L)}{dL} = \frac{d(Q(L)/L)}{dL} = \frac{L.Q'(L) - Q(L)(1)}{L^2} = \frac{MP_L - AP_L}{L}$$

At $\max_L AP_L$, $\frac{d(AP_L)}{dL} = 0$

⇨ $\frac{MP_L - AP_L}{L} = 0$

⇨ $MP_L = AP_L$ at the maximum of AP(L).

d. (4 points) Given that the firm faces the demand function

Q = 100 - 2P

derive the marginal revenue product (*MRP*) function.

$Q=100-2P \Rightarrow P=50-0.5Q$

$TR(Q)=P(Q)xQ = 50Q-0.5Q^2 \Rightarrow MR(Q)=50-Q$

$$MRP = \frac{d(TR)}{dL} = \frac{d(50-Q^2)}{dL} = \frac{dTR}{dQ} \cdot \frac{dQ}{dL} = (50 - Q)\frac{d(6L^2 - L^3)}{dL}$$

$$MRP = (50 - 6L^2 + L^3)(12L - 3L^2) = 3(200L - 50L^2 - 24L^3 + 10L^4 - L^5)$$