

EE325 Section 1 HW 2 Due Thursday February 20<sup>th</sup> (23:00 hr.), 2020

Use 4 decimal places for numerical answers

1. In Table 1.a.  $X_i$  is total microeconomics exam point (total points are 100) and  $Y_i$  is GPA of each student.

**Table 1.a**

Student	$Y_i$	$X_i$
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$y - \bar{y}$	$x - \bar{x}$
-0.4125	-14.625
0.1875	5.625
-0.2125	0.125
0.2875	2.375
0.3375	9.375
-0.2125	-2.625
-0.3125	-2.625
-0.4375	11.875

$$\bar{y} = \frac{2.8 + 3.4 + 3 + 3.5 + 3.6 + 3.0 + 2.7 + 3.7}{8}$$

$$= 3.2125$$

$$\bar{x} = \frac{63 + 72 + 78 + 81 + 87 + 75 + 75 + 90}{8}$$

$$= 77.625$$

$$= 77.625$$

1.1 Now consider the two-variable  $Y_i = \beta_0 + \beta_1 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$ . Use OLS to find the estimator of  $\beta_0$  and  $\beta_1$ . (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{19.4375}{511.875} = 0.0379 \neq$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= 3.2125 - (0.0379)(77.625)$$

$$= 0.5181$$

1.2 For each observation  $i$ , find  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum_{i=0}^N \hat{u}_i = 0$ .

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 \bar{x}$$

$$\hat{u}_i = y_i - \hat{y}_i$$

$\hat{y}_1 = 2.8143$	$\hat{u}_1 = 0.0857$
$\hat{y}_2 = 3.0209$	$\hat{u}_2 = 0.3911$
$\hat{y}_3 = 3.2273$	$\hat{u}_3 = -0.2253$
$\hat{y}_4 = 3.3275$	$\hat{u}_4 = 0.1725$
$\hat{y}_5 = 3.5319$	$\hat{u}_5 = 0.0681$
$\hat{y}_6 = 3.7231$	$\hat{u}_6 = -0.7231$
$\hat{y}_7 = 3.9231$	$\hat{u}_7 = -0.4231$
$\hat{y}_8 = 3.6341$	$\hat{u}_8 = 0.0659$

It's true that  $\sum_{i=0}^N \hat{u}_i = 0$

1.3 Find  $var(\hat{u}_i)$ ,  $var(\hat{\beta}_0)$ ,  $var(\hat{\beta}_1)$

$$var(\hat{\beta}_0) = \frac{\sum x_i^2}{n \sum (x_i - \bar{x})^2} \sigma^2$$

2. Data is listed in the table

$X_i$	$Y_i$	$y_i - \bar{y}$	$x_i - \bar{x}$
10	0	-9.1	-10
12	2	-7.1	-8
14	5	-4.1	-6
16	6	-3.1	-4
18	7	-2.1	-2
22	10	0.9	2
24	10	0.9	4
26	15	5.9	6
28	16	6.9	8
30	20	10.9	10

$$\bar{y} = \frac{0+2+5+(7+6+6+15+16+20)}{10} = 9.1$$

$$\bar{x} = \frac{10+12+14+16+18+22+24+26+28+30}{10} = \frac{200}{10} = 20$$

2.1 From the simple regression model  $Y_i = \beta_0 + \beta_1 X_i + u_i$ ,  $u_i \sim NIID(0, \sigma^2)$ . Find estimators of  $\beta_0$  and  $\beta_1$  from the OLS method and interpret the meaning.

find  $\hat{\beta}_0$  and  $\hat{\beta}_1$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^n (y_i - \bar{y})(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\hat{\beta}_0 = 9.1 - (0.9955 \times 20)$$

$$= -8.81 \neq$$

$$= \frac{91 + 86.9 + 24.6 + 18.4 + 4.2 + 2.8 + 3.6 + 35.4 + 55.2 + 109}{100 + 64 + 36 + 16 + 4 + 4 + 16 + 36 + 64 + 100}$$

$$= \frac{394}{440} = 0.8955$$

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 \bar{x} \quad \hat{u}_i = y_i - \hat{y}_i$$

2.2 Find the value of  $\hat{Y}_i$  and  $\hat{u}_i$ . Show that  $\sum_{i=0}^N \hat{u}_i = 0$ .

0.1455	-0.1455
1.9364	0.0326
7.7276	1.2229
5.5192	0.4894
9.3091	-0.5071
10.6909	-0.9909
12.6919	-2.6919
14.4929	0.5223
11.2830	-0.2830
19.0546	1.9453

2.3 Plot graph and draw regression line. Does the line pass  $(\bar{X}, \bar{Y})$ ? (20, 9.1)

$$y = 1.5x + c$$

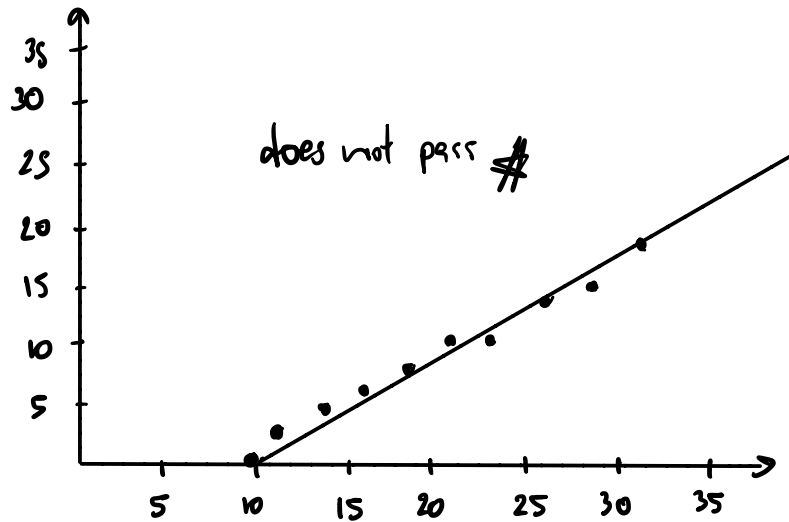
$$5 = 1.5 \times 14 + c$$

$$c = 16$$

$$y = 1.5x - 16$$

$$9.1 = 1.5(20) - 16$$

$\neq$



2.4 If  $X_i = 16$ , what is the predicted Y?

$$E(y | x = 16) = 5.5192$$

2.5 Find  $var(\hat{u}_i)$ ,  $var(\hat{\beta}_0)$ ,  $var(\hat{\beta}_1)$

$$\text{unbiased } E(\hat{\beta}_1) = \beta_1$$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where  $u_i \sim NIID(0, \sigma^2)$ . Find an OLS estimator of  $\beta_1$ . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$u_i = y_i - \beta_1 x_i$$

denoted the  
summation  
square as

$$f(\beta_0, \beta_1) = \sum_{i=1}^n (y_i - \beta_1 x_i)^2$$

F.O.C. derivative w.r.t.  $\beta_1 = 0$

$$0 = -2 \sum_{i=1}^n (y_i - \beta_1 x_i) x_i$$