Assignment 4

DUE DATE: Tuesday 9^{nd}, March 2021.

I pledge to the Honor Code and to obey all rules for taking and performing homework assignments as specified by the course instructor.

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Question 1 (50 points) Your score.....................

Given the daily log returns : (R_t) can be explained by the AR(2) model as following:

$$(1 - 1.5B + 0.9B^2)R_t = 0.25 + \varepsilon_t$$

where ε_t is distributed as the Gaussian White Noise with mean $(\mu) = 0$ and variance (σ^2) =0.25
B lag-operator

Question 1.1 (10 points) Your score.....................

$(1 - 1.5B + 0.9B^2)R_t = 0.25 + \varepsilon_t$

From the above AR(2) model, Is the model weakly stationary? Write down the reverse characteristic equation and find out the conditions to support your answer.

Revers character equation.

$r^2 - 1.5R + 0.9 = 0$

$\lambda^2 - \phi_1\lambda - \phi_2 = 0$

$\lambda_i = \dfrac{\phi_1 \pm \sqrt{\phi_1^2 + 4\phi_2}}{2}$

$\lambda = \dfrac{\phi_1 \pm \sqrt{\phi_1^2 - 4(1)(-\phi_2)}}{2}$

$\Rightarrow \lambda_i = \dfrac{(-1.5) \pm \sqrt{(1.5)^2 - 4(-0.9)}}{2}$

$= \dfrac{1.5 \pm \sqrt{(2.25) + (3.6)}}{2}$

$= \dfrac{1.5 \pm 2.4186}{2}$

$\Rightarrow \dfrac{3.918677}{2}, \dfrac{0.8186773245}{2}$

$\underbrace{1.959}_{r_1}, \underbrace{0.459}_{r_2}$

since $r > 1$, $r_2 < 1$
so, it is not weakly stationary.

Question 1.2 (10 points) Your score......................

$(1-1.5B+0.9B^2)R_t = 0.25+\varepsilon_t$ take $\varepsilon_t = 0$.

Calculate the unconditional mean: $E(R_t)$ of R_t and the conditional mean: $E(R_t|F_{t-1})$

$R_t = 1.5R_{t-1} - 0.9R_{t-2} + a_t + 0.25$

① $E(R_t) - 1.5E(R_{t-1}) + 0.9E(R_{t-2}) = 0.25$

⊕ By $E(R_t)$ of R_t so we get:

$R_t - 1.5R_t + 0.9R_t = 0.25$

$0.4R_t = 0.25$

$\Rightarrow R_t = \frac{0.25}{0.4} = \underline{0.625}$

②. $E(R_t|F_{t-1}) - 1.5E(R_{t-1}|F_{t-1}) + 0.9E(R_{t-2}|F_{t-1}) = 0.25$

⊕. By $E(R_t|F_{t-1})$ of R_t so we get:

$R_t - 1.5R_t + 0.9R_t = 0.25$

$0.4R_t = 0.25$

$\boxed{R_t = 0.625}$

Question 1.3 (10 points)

Your score......................

$(1 - 1.5B + 0.9B^2)R_t = 0.25 + \varepsilon_t$

Find out the unconditional variance: $Var(R_t)$ of R_t and conditional variance $Var(R_t|F_{t-1})$ of R_t

$$R_t = 0.25 + \varepsilon_t + 1.5(R_{t-1}) - 0.9(R_{t-2})$$

$$R_t = 1.5R_{t-1} - 0.9R_{t-2} + a_t + 0.25$$

$$Var(R_t) = (1.5)^2 Var(R_t) - (0.9)^2 Var(R_t) + \sigma_a^2 + 2Cov(1.5R_{t-1}R_{t-2}) + 2Cov(1.5R_{t-1}a_t) + 2Cov(0.9R_{t-2}a_t)$$

$$\Rightarrow Var(R_t) = 2.25Var(R_t) - 0.81Var(R_t) + \sigma_a^2 + 2Cov(1.5R_{t-1}R_{t-2}) + 2Cov(1.5R_{t-1}a_t) + 2Cov(0.9R_{t-2}a_t)$$

Question 1.4 (10 points) Your score......................

$$(1-1.5B+0.9B^2)R_t = 0.25+\varepsilon_t$$

Calculate the autocorrelation: ρ_l for l=1 and 2 of R_t. Also, write down the autocorrelation: ρ_l when l $\geq$ 2.

$$R_t = 1.5R_{t-1} - 0.9R_{t-2} + 0.25 + a_t$$

$$E[r_t r_{t-\ell}] = 1.5E(r_{t-1}r_{t-\ell}) - 0.9E(r_{t-2}r_{t-\ell}) + E[a_t r_{t-\ell})$$

$\ell = 1$, $$E[r_t r_{t-1}] = 1.5E[r_{t-1}r_{t-1}] - 0.9E[r_{t-2}r_{t-1}] + \underbrace{E[a_t r_{t-1}]}_{=0}$$

$$\boxed{= 1.5\gamma_0 - 0.9\gamma_1 + 0}$$

$\ell_2 \Rightarrow$ $$E[r_t r_{t-2}] = 1.5E[r_{t-1}r_{t-2}] - 0.9E[r_{t-2}r_{t-2}] + \underbrace{E[a_t r_{t-2}]}_{0}$$

$$\boxed{= 1.5\gamma_2 - 0.9\gamma_3 + 0}$$

Question 1.5 (10 points) | Your score......................

$$(1 - 1.5B + 0.9B^2)R_t = 0.25 + \varepsilon_t$$

Given $R_{1000} = 0.01$ $R_{999} = 0.02$ $R_{998} = 0.03$ $\varepsilon_{1000} = -0.01$ $\varepsilon_{999} = -0.02$ $\varepsilon_{998} = -0.03$ Obtain 1-step, 2-setp 95 % interval forecasts for R_t at the forecast origin t = 1000. Also the ∞-step 95 % interval forecasts for R_t. Draw these intervals.

(+) Step 1

$R_t = 1.5 R_{t-1} - 0.9 R_{t-2} + a_t + 0.25$

$\hat{R}_{1001} = 0.25 + 1.5(R_{1000}) - 0.9(R_{999}) + \varepsilon_{1001}$

$= 0.25 + 1.5(0.01) - 0.9(0.02) + 0$

$= 0.25 + 0.015 + 0.018 + 0$

$\boxed{= 0.283}$

(+) $e_h = \varepsilon_{1001}$

$Var(e_h(1)) = \sigma_a^2$

$$\Rightarrow \boxed{\begin{matrix} 0.283 - 1.96(\sigma_a) \\ 0.283 + 1.96(\sigma_a) \end{matrix}}$$

1 Step a head : $(0 \pm 1.96 \sqrt{\sigma_a^2})$

$$(1-1.5B+0.9B^2)R_t = 0.25+\varepsilon_t$$

⊕ Step 2

$$r_{h+2} = \phi_0 + \phi_1 r_{h+1} + \phi_2 r_{h+2} + \dots + a_{h+2}$$

$$R_t = 1.5 R_{t-1} - 0.9 R_{t-2} + 0.25 + a_t$$

$$\hat{R}_{t_{1002}} = 0.25 + 1.5(R_{1001}) - 0.9(R_{1000}) + \varepsilon_{1002}$$

$$= 0.25 + 1.5(0.283) - 0.9(0.01)$$

$$= 0.25 + 0.4245 - 0.009$$

$$\boxed{= 0.6655}$$

$$Var(e_2) = \phi_1^2 \sigma_a^2 + \sigma_a^2$$

$$= (1+\phi_1^2)\sigma_a^2$$

$$= (1+(1.5)^2)\sigma_a^2$$

$$= (1+2.25)\sigma_a^2$$

$$\boxed{= 3.25\sigma_a^2}$$

$\Rightarrow$ $0.6655 + 1.96(3.25\sigma_a^2)$

$0.6655 - 1.96(3.25\sigma_a^2)$

⊕ mean equal to 0

Second step ahead : $\boxed{(0 \pm 1.96\sqrt{3.25\sigma_a^2})}$