BACHELOR of ECONOMICS
International Program

Thammasat University | Thailand

FN 312 **Investments**

Option Valuation 2: The Black Scholes model

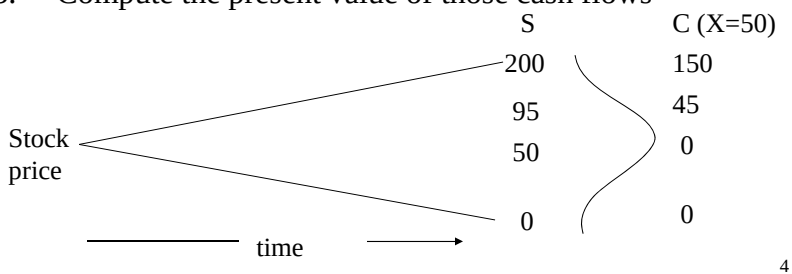
Road Map/key Ideas

- How to price options using the Black-Scholes (1973)
- Assumptions in the binomial model and B-S
- Implied volatility

3

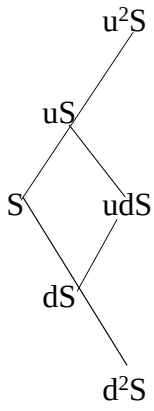
Methodology

1. Assume a stochastic process of the underlying asset (i.e., determine what are the possible future outcomes of the price of the underlying asset and the probabilities of those outcomes.)
2. Compute the cash flow that the holder of the option will obtained under the different outcomes
3. Compute the present value of those cash flows



4

Black-Scholes Option Pricing



Binomial → lognormal (Black Scholes)

5

Black-Scholes Option Pricing

$$C_0 = S_0 N(d_1) - Xe^{-rT} N(d_2)$$

$$P_0 = Xe^{-rT} N(-d_2) - S_0 N(-d_1)$$

Where

$$d_1 = \frac{\mathrm{Ln}(S_0 / X) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$d_2 = d_1 - \sigma\sqrt{T}$

r = risk-free rate (annualized, continuously compounded)

T = time to maturity in years

σ = standard deviation of annualized continuously compounded rate of return on stock in decimal

6

Black-Scholes Option Pricing

N(.) = Cumulative Standard Normal distribution

Ln(.) = Natural logarithm

- Put price can also be obtained from put-call parity.

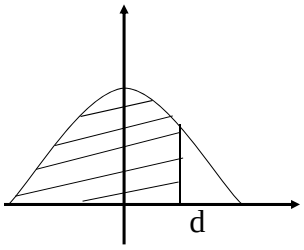
Recall from Statistics

Suppose Z is standard normal then

$P[Z \leq d] = N(d)$

7

Black-Scholes Option Pricing



To get N(d) you can use the table of cumulative normal distribution or the function NORMSDIST in excel.

8

Example

Suppose S_0 = \$50 X = \$45

r = 6% σ = 20% T = 0.25 years

Calculation:

$$d_1 = \frac{\ln(\frac{50}{45}) + (0.06 + \frac{(0.20)^2}{2})\,0.25}{(0.20)\sqrt{0.25}}$$

$$= 1.2536$$

$$d_2 = 1.2536 - \sigma\sqrt{T} =$$

$$= 1.2536 - 0.1 = 1.1536$$

9

Example

$N(d_1) = 0.8950$

$N(d_2) = 0.8757$

$C = 50(0.8950) - 45(0.9851)\,(0.8757)$

S $N(d_1)$ X e^{-rT} $N(d_2)$

$= 44.7500 - 38.8193$

$= 5.9307 \cong \$5.93$

10

Binomial Tree VS B-S

		BS price	Binomial price	N
stock price	100	3.7146	3.0147	2
strike price	100	3.7146	3.347	4
volatility	0.15	3.7146	3.466	6
T	1 year	3.7146	3.563	10
Treasury	0.05	3.7146	3.663	30
Put option price ?		3.7146	3.699	100
		3.7146	3.7115	500

11

Binomial Tree VS B-S

- B-S easy to use quite accurate for European option, especially ATM options
- B-S cannot be used to value American options that have optimal early exercise while binomial tree can be modified to value American options.
- Binomial tree method requires more programming and longer execution time for accuracy
- Binomial method is easy to modify for pricing exotic options

12

Assumptions in Binomial model & B-S

- Perfect Capital Markets (e.g., short-selling allowed, no transaction costs)
- Stock prices are lognormal. Stock Returns are normally distributed, i.e., stock returns are random walks
- The underlying asset is continuously tradable
- Future volatility is known (non-stochastic)

13

Concept check

You are trying to price a contract to mine a gold mine. The contract lasts 5 years. You can only mine it once and get 100K tons of gold. The cost of mining it is fixed at 10M. The revenue depends on the market price of gold. Can you use the option valuation framework to price this contract? If yes, European or American, what is X?

G) Yes. American, X=10M

Y) Yes. European, X<10M

R) No.

14

Implied Volatility

- Is the volatility that makes the option price calculated using the B-S model equal to the the market price of the option.

- How to obtain it?

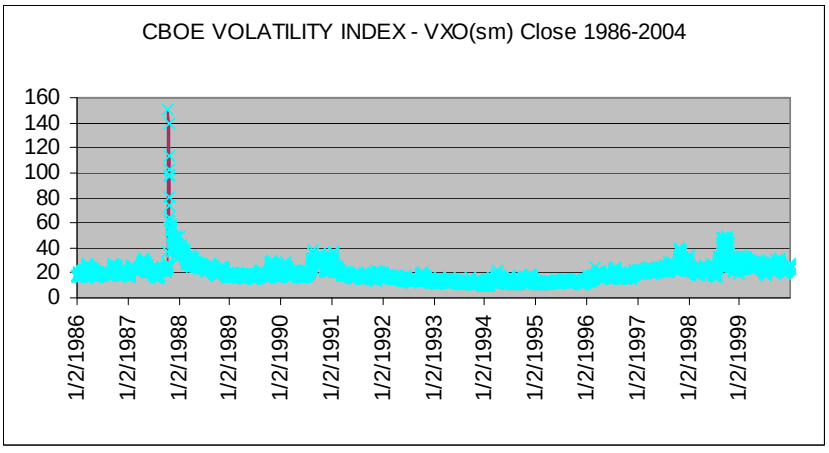
Solve the non-linear B-S pricing equation for the volatility given an option price

- Why do we care?

You want to know what the market consensus is for future volatility

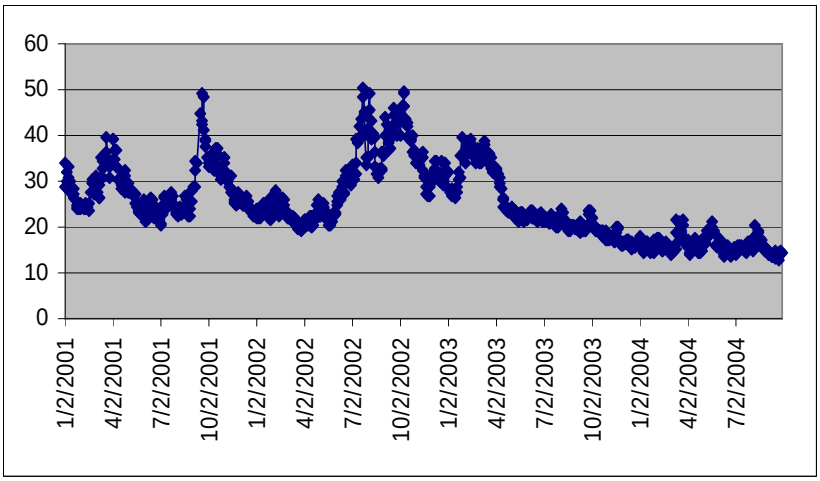
15

Implied Volatility



16

Implied Volatility



17

Concept check

Option of the same stock, same maturity date, same strike price, A with higher implied volatility than B, which one is more richly priced?

G) A

R) B

18

Summary

- How to price options using the Black-Scholes (1973)
- Assumptions in the binomial model and B-S
- Implied volatility

19