

The Log-Linear Model

$$\ln Y = \beta_1 + \beta_2 \ln X$$

$$\frac{1}{Y} \frac{dY}{dX} = \frac{\beta_2}{X} \frac{dX}{dX}$$

$$\beta_2 = \frac{X}{Y} \frac{dY}{dX} = \frac{dY/Y}{dX/X} = \text{elasticity}$$

Semilog models

$$\ln Y = \beta_1 + \beta_2 X$$

$$\frac{1}{Y} \frac{dY}{dX} = \beta_2$$

$$\beta_2 = \frac{dY/Y}{dX}$$

$$\beta_2 = \frac{\text{relative change in regressand}}{\text{absolute change in regressor}}$$

β_2 is known as the semielasticity of Y with respect to X

$100 * \beta_2$ is known as the semielasticity of Y with respect to X

$$Y = \beta_1 + \beta_2 \ln X$$

$$\frac{dY}{dX} = \beta_2 \frac{1}{X}$$

$$\beta_2 = \frac{dY}{dX / X}$$

The absolute change in Y for a percentage change in X

If $\Delta X / X$ changes by 0.01 unit or 1%, the absolute change in Y is $0.01 * \beta_2$

Reciprocal model

$$Y_i = \beta_1 + \beta_2 \left(\frac{1}{X} \right)$$

$$\frac{dY}{dX} = -\beta_2 \left(\frac{1}{X^2} \right)$$

if β_2 is positive, the slope is negative throughout

β_2 is negative, the slope is positive throughout