EE 320 Introductory Mathematical Economics

Semester 2/2012

Problem Set 6 – Suggested Answers

Derivatives of More-Than-One Independent Variable Function[1]

1. $\partial M/\partial Y = 0.14 > 0$: Money demand increases with income.
   $\partial M/\partial r = -0.84 \cdot 76.03(r-2)^{-1.84} = -63.87(r-2)^{-1.84} < 0$: Money demand decreases as the interest rate increases.

2. $D'_p(p,q) = bq^{-\alpha} < 0$, showing that the demand decreases as its own price increases.
   $D'_q(p,q) = -b\alpha pq^{-\alpha-1} > 0$, showing that the demand increases as the price of a competing product increases.

3. $\partial T/\partial x = \frac{ky}{d^n} > 0$ and $\partial T/\partial y = \frac{kx}{d^n} > 0$, suggesting that the number of travelers increases as the size of either city increases.
   $\partial T/\partial d = -\frac{nkxy}{d^{n+1}} < 0$, suggesting that the number of travelers decreases as the distance between the two cities increases.

4. $\frac{\partial}{\partial m}\left(\frac{pD(p,m)}{m}\right) = p\frac{mD_m - D}{m^2} = \frac{pD}{m^2}\left[m\frac{\partial D}{\partial m}\cdot\frac{1}{D} - 1\right] = \frac{pD}{m^2}[\varepsilon_m - 1] > 0 \; iff \; \varepsilon_m > 1.$
   So, *pD/m* increases with *m* if $\varepsilon_m > 1$ ("luxury" goods).
   Similarly, one can show that $\frac{\partial}{\partial m}\left(\frac{pD(p,m)}{m}\right) = \frac{pD}{m^2}[\varepsilon_m - 1] < 0 \; iff \; \varepsilon_m < 1$ ("necessity" goods).

5. (a) $\partial Y/\partial K \approx 0.083K^{0.356}L^{0.562}$ and $\partial Y/\partial S \approx 0.035K^{1.356}L^{-0.438}$.
   (b) If K and S are both doubled, the catch becomes $2^{1.356+0.562} = 2^{1.918} \approx 3.779$ times higher.

6. Suppose that a firm produces units of commodity using L units of labor. $Pf'(L) - w = 0$
   (a) Profit function: $(L) = Pf(L) - wL$.
   F.O.C. $Pf'(L) - w = 0 \Rightarrow \frac{P}{2\sqrt{L^*}} = w.$
   Thus, the FOC in term of $L^*(w, P)$ is $Pf'[L^*(w,P)] - C'_L[L^*(w,P)] = 0.$

[1] All questions in this problem set are from Sydsaeter and Hammond, 2008.

(b) $\partial L^*/\partial P = \frac{f'(L^*)}{C''_{LL}(L^*,w) - pf''(L^*)}$; $\quad\quad$ $\partial L^*/\partial w = \frac{C''_{Lw}(L^*,w)}{pf''(L^*) - C''_{LL}(L^*,w)}$.

7. Take logarithm on both sides and differentiate with respect to K:

$$\frac{c}{y}\frac{\partial y}{\partial K} lny + (1 + clny)\frac{\partial y}{\partial K} = \frac{\alpha}{K}.$$ Then solve for $\frac{\partial y}{\partial K}$.

$\frac{\partial y}{\partial L}$ can be derived in the same manner.

8. $\frac{dX}{dN} = g(u) + Ng'(u)\frac{du}{dN} = g(u) + g'(u)(\varphi'(N) - u),$ where $u = \frac{\varphi(N)}{N}$

$$\frac{d^2X}{dN^2} = g'(u)\frac{du}{dN} + g''(u)\frac{du}{dN}(\varphi'(N) - u) + g'(u)\left(\varphi''(N) - \frac{du}{dN}\right)$$

$$= \frac{1}{n}g''\left(\frac{\varphi(N)}{N}\right)\left[\varphi'(N) - \frac{\varphi(N)}{N}\right]^2 + g'\left(\frac{\varphi(N)}{N}\right)\varphi''(N).$$

9. (a) $MRS_{yx} = \frac{U_x}{U_y} = \frac{2y}{3x}$

(b) $MRS_{yx} = \frac{U_x}{U_y} = \frac{y}{x+1}$

(c) $MRS_{yx} = \frac{U_x}{U_y} = \left(\frac{y}{x}\right)^3$

10. (a) $\partial p/\partial w = \frac{L}{F(L)}$; $\quad\quad$ $\partial p/\partial B = \frac{1}{F(L)}$;

$\partial L/\partial w = \frac{F(L) - LF'(L)}{pF(L)F''(L)}$; $\quad\quad$ $\partial L/\partial B = -\frac{F'(L)}{pF(L)F''(L)}$

(b) Since $p > 0$, $F'(L) > 0$, and $F''(L) < 0$, $\partial p/\partial w > 0$; $\partial p/\partial B > 0$; $\partial L/\partial B > 0$.

We can show that $F(L) - LF'(L) = B/p > 0$. Thus, $\partial L/\partial w < 0$.