B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics (Section 046402)

Semester 1/2013

Quiz 4 (a)

1. (3 point) Find total differential of $y = (2x_1)(x_1 + x_2)(2x_3^2 - 1)$.

Let $u = 2x_1$, $v = x_1 + x_2$, $w = 2x_3^2 - 1$

$\Rightarrow du = 2dx_1$, $dv = dx_1 + dx_2$, $dw = 4x_3\,dx_3$

$dy = uv\,dw + uw\,dv + vw\,du$

$= (2x_1)(x_1+x_2)(4x_3\,dx_3) + (2x_1)(2x_3^2-1)(dx_1+dx_2) + (x_1+x_2)(2x_3^2-1)(2dx_1)$

$= \left[(2x_1+2x_1+2x_2)(2x_3^2-1)\right]dx_1 + \left[(2x_1)(2x_3^2-1)\right]dx_2 + \left[(8x_1x_3)(x_1+x_2)\right]dx_3$

$= \underbrace{\left[(4x_1+2x_2)(2x_3^2-1)\right]}_{\frac{\partial y}{\partial x_1}}dx_1 + \underbrace{\left[(2x_1)(2x_3^2-1)\right]}_{\frac{\partial y}{\partial x_2}}dx_2 + \underbrace{\left[(8x_1x_3)(x_1+x_2)\right]}_{\frac{\partial y}{\partial x_2}}dx_3$

2. (3 points) Find the total derivative of y w.r.t. x when

$y = f(u, v, x) = u^3 - 2\sqrt{v} + x^2$ where $u = g(x)$ and $v = h(x)$.

$\frac{dy}{dx} = \frac{\partial f}{\partial u}\cdot\frac{du}{dx} + \frac{\partial f}{\partial v}\cdot\frac{dv}{dx} + \frac{\partial f}{\partial x}$

$= 3u^2\cdot g'(x) - (2)\left(\frac{1}{2}\right)v^{-1/2}\cdot h'(x) + 2x$

$= 3[g(x)]^2\cdot g'(x) - \frac{h'(x)}{\sqrt{h(x)}} + 2x$.

3. (4 points) Given $F(z, x, y) = x^2 + y^2 + 2z^2 + xz - 3xy$, use the implicit function rule to determine $\frac{\partial z}{\partial x}$ and $\frac{\partial y}{\partial x}$.

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{2x + z - 3y}{4z + x}$$

$$\frac{\partial y}{\partial x} = -\frac{F_x}{F_y} = -\frac{2x + z - 3y}{2y - 3x}$$