Chapter 1 , Section 2.6

Q.6 • given that a demand = 1500 when $p = \$1$

$\therefore \quad p = D(x) \Rightarrow 1 = D(1500)$

estimate that for each 5 cent ↑ in price, D ↓ by 25

↳ linear drop of D when p increase linearly

$\therefore \; p = D(x)$ = linear function = $mx + b$

↑ price function

$$1 = m(1500) + b \quad \text{—— (1)}$$

$$1.05 = m(1475) + b \quad \text{—— (2)}$$

(2) − (1)

$$0.05 = m(-25)$$

$$m = -0.002$$

insert $m = -0.002$ in eq. (1) $\quad 1 = -0.002(1500) + b$

$$b = 1 + 3 = 4$$

$$\therefore \quad \boxed{p(x) = -0.002x + 4}$$

Check when 10 cent ↑ in price ⇒ D ↓ by 25×2 = 50

$$1.1 = -0.002(x) + 4$$

$x = 1450$ which is 1500 − 50

O.K.

• Find C(x)

$$C(x) = 180 + 0.3x$$

C(x): cost per month

180: overhead per month

0.3: labour + material cost per unit

x: units per month

$\underbrace{\text{labour + material cost per unit} \cdot x \text{ units per month}}_{\text{labour + material cost per month}}$

Profit = Revenue − Cost

$= p(x) \cdot x - C(x)$

$= (-0.002x + 4)x - (180 + 0.3x)$

$= -0.002x^2 + 4x - 180 - 0.3x$

$P(x) = -0.002x^2 + 3.7x - 180$

profit per month as a function of no of units sold per month

However, the question asks to find profit as a function of price

price, $p = -0.002x + 4$

$\therefore \dfrac{p-4}{-0.002} = x$

$\therefore P(p) = -0.002\left(\dfrac{p-4}{-0.002}\right)^2 + 3.7\left(\dfrac{p-4}{-0.002}\right) - 180$

profit price

$= \dfrac{\cancel{-0.002}\,(p-4)^2}{(-0.002)\cancel{(-0.002)}} + \dfrac{3.7}{-0.002}(p-4) - 180$

$= -500(p^2 - 8p + 16) - 1850(p-4) - 180$

$= -500p^2 + 4000p - 8000 - 1850p + 7400 - 180$

$= -500p^2 + 2150p - 780$

the question calls price p, x

$\therefore P(x) = -500x^2 + 2150x - 780$

The plot of $P(x)$ will be a parabola $\Rightarrow$ try to express function in a vertex form

$$P(x) = -500\,(x^2 - 4.3x - 1.56)$$

$$= -500\,(x-h)^2 + k$$

$$(x-h)^2 = x^2 - 2hx + h^2$$

$$-2hx = 4.3 \Rightarrow h = 2.15$$

Since $a = -500$ (Reminder: $y = a(x-h)^2 + k$ = vertex form) means the parabola opens downward with a maximum at point (h, k)

$\therefore$ the value of x resulting in the maximum profit will be equal to h which is 2.15