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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

$$\text{a. } \sum_{i=1}^5 (a + bx_i) = 5a + b \sum_{i=1}^5 x_i \\ = 5a + b(x_1 + x_2 + \dots + x_5)$$

$$\text{b. } \sum_{y=0}^5 f(x+y) = f(x+0) + f(x+1) + f(x+2) + f(x+3) + f(x+4) + f(x+5)$$

$$\text{c. } \sum_{i=1}^{10} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10(10+1)(2(10)+1)}{6} = 385$$

$$\text{d. } \sum_{x=1}^2 \sum_{y=2}^3 (2x+y) = \sum_{x=1}^2 [2x+2+3] = \sum_{x=1}^2 2x+5 = 2(1+2)+5+5 = 11$$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

a. Find the value of b

$$f(x) = P(X=x) = 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b = 1 \\ 8b = 1 \\ b = \frac{1}{8}$$

b. Find the answer for $P(X \leq 2)$

$$P(X \leq 2) = 1 - P(X > 2) \\ = 1 - P(X=3) - P(X=4) \\ = 1 - (0.5)(\frac{1}{8}) - 0.25(\frac{1}{8}) = 0.9625$$

c. Find the answer for $P(-2 \leq X \leq 3)$

$$P(-2 \leq X \leq 3) = 1 - P(X < -2) - P(X > 3) \\ = 1 - (0.5)(\frac{1}{8}) - 0.25(\frac{1}{8}) \\ = 0.9625$$

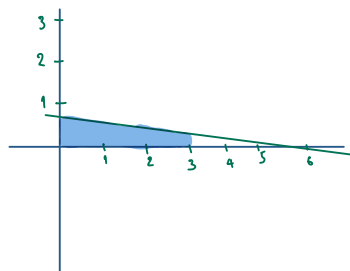
d. Find the answer for $P(X \geq 1)$

$$P(X \geq 1) = 1 - P(X < 1) \\ = 1 - P(X=0) - P(X=-1) - P(X=-2) \\ = 1 - (2.25)(\frac{1}{8}) - (1)(\frac{1}{8}) - 0.5(\frac{1}{8}) \\ = 0.53125$$

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is $(x, y) = (6, \frac{2}{3})$

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

- a. Plot graph for $f(x)$



- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} P(1 \leq X \leq 3) &= \int_1^3 f(x) dx \\ &= \int_1^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x \right) dx \\ &= \left[-\frac{(3)^3}{27} + \frac{6(3)}{9} \right] - \left[-\frac{(1)^3}{27} + \frac{6(1)}{9} \right] = -\frac{9}{18} + \frac{16}{9} + \frac{1}{18} - \frac{6}{9} = \frac{-6}{18} + \frac{12}{9} = \frac{16}{18} \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$\begin{aligned} P(X \geq 2) &= \int_2^3 f(x) dx \\ &= \int_2^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x \right) dx \\ &= \left[-\frac{(3)^3}{27} + \frac{6(3)}{9} \right] - \left[-\frac{(2)^3}{27} + \frac{6(2)}{9} \right] = \frac{7}{18} \end{aligned}$$

- d. Find the expected value of X

$$\begin{aligned} E(X) &= \int_{-\infty}^{\infty} x f(x) dx \\ &= \int_0^3 x \left(-\frac{1}{9}x + \frac{6}{9} \right) dx \\ &= \int_0^3 \left(-\frac{1}{9}x^2 + \frac{6}{9}x \right) dx \\ &= \left[-\frac{x^3}{27} + \frac{6x^2}{18} \right]_0^3 \\ &= \left[-\frac{(3)^3}{27} + \frac{6(3)^2}{18} \right] - \left[-\frac{(0)^3}{27} + \frac{6(0)^2}{18} \right] \\ &= -1 + 3 \\ &= 2 \end{aligned}$$

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

- a. Construct the joint probability distribution function (PDF) table of

X and Y

x/y	1	2	3	4	5	6	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{2}$
	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	1

- b. Find the marginal probability distribution function (PDF) of X

The marginal prob of x $p(x=x)$ is represented in green

- c. Find the marginal probability distribution function (PDF) of Y

The marginal prob of y $p(y=y)$ is represented in pink

- d. Find the conditional probability distribution function (PDF) of

X given Y is equal to 1

x	$P(x=x)$
1	$\frac{1}{6}$
2	$\frac{1}{6}$
3	$\frac{1}{6}$
4	$\frac{1}{6}$
5	$\frac{1}{6}$
6	$\frac{1}{6}$

- e. Find the expected value of X given Y is equal to 1

$$E(X|y=1) = \sum x_i P(x=x_i | y=1) = \frac{\sum x_i P(x=x_i, y=1)}{P(y=1)} = \frac{1}{P(y=1)} \sum x_i P(x=x_i, y=1)$$

$$= \frac{1}{0.5} \left[(1 \cdot \frac{1}{12}) + (2 \cdot \frac{1}{12}) + (3 \cdot \frac{1}{12}) + (4 \cdot \frac{1}{12}) + (5 \cdot \frac{1}{12}) + (6 \cdot \frac{1}{12}) \right] = \frac{7}{2}$$

- f. Find the variance of X given Y is equal to 1

$$V(X|y=1) = \sum (x - E(X|y=1))^2 \cdot P(x|y=1)$$

$$= \left(\frac{25}{4} \cdot \frac{1}{6} \right) + \left(\frac{9}{4} \cdot \frac{1}{6} \right) + \left(\frac{1}{4} \cdot \frac{1}{6} \right) + \left(\frac{1}{4} \cdot \frac{1}{6} \right) + \left(\frac{9}{4} \cdot \frac{1}{6} \right) + \left(\frac{25}{4} \cdot \frac{1}{6} \right)$$

$$= \frac{120}{24} = \frac{10}{3}$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{N} \sum_{i=1}^3 X_i\right) \\ &= \frac{1}{N} E(X_1 + X_2 + X_3) \\ &= \frac{1}{N} [E(X_1) + E(X_2) + E(X_3)] \\ &= \frac{1}{3} [\mu_X + \mu_X + \mu_X] \\ &= \frac{1}{3} \cdot 3 \mu_X = \mu_X = \bar{X} \end{aligned}$$

$$\begin{aligned} \text{Var}(\bar{X}) &= \text{Var}\left(\frac{1}{N} \sum_{i=1}^3 X_i\right) \\ &= \frac{1}{N^2} \text{Var}(X_1 + X_2 + X_3) \\ &= \frac{1}{N^2} [\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3)] \\ &= \frac{1}{N^2} \end{aligned}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$\begin{aligned} E(\bar{X}) &= E\left(\frac{1}{N} \sum_{i=1}^4 X_i\right) \\ &= \frac{1}{N} E(X_1 + X_2 + X_3 + X_4) \\ &= \frac{1}{N} [E(X_1) + E(X_2) + E(X_3) + E(X_4)] \\ &= \frac{1}{4} [\mu_X + \mu_X + \mu_X + \mu_X] \\ &= \frac{1}{4} \cdot 4 \mu_X = \mu_X \end{aligned}$$

$$\begin{aligned} \text{Var}(\bar{X}) &= \text{Var}\left(\frac{1}{N} \sum_{i=1}^4 X_i\right) \\ &= \frac{1}{N^2} \text{Var}(X_1 + X_2 + X_3 + X_4) \\ &= \frac{1}{N^2} [\text{Var}(X_1) + \text{Var}(X_2) + \text{Var}(X_3) + \text{Var}(X_4)] \\ &= \frac{1}{4^2} \cdot 4 \sigma^2 = \frac{\sigma^2}{4} = 0.25 \sigma^2 \end{aligned}$$

b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show

that $\tilde{X}$ is an unbiased estimator of μ

$$\tilde{X} = \frac{1}{4}(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$E(\tilde{X}) = E\left(\frac{1}{4} \sum_{i=1}^4 X_i\right)$$

$$= \frac{1}{4} E(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$= \frac{1}{4} [0.5E(X_1) + E(X_2) + 0.5E(X_3) + 2E(X_4)]$$

$$= \frac{1}{4} (0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x)$$

$$= \frac{1}{4} \cdot 4\mu_x = \mu_x$$

$\tilde{X}$ is unbiased estimator of μ

$$\text{Var}(\tilde{X}) = \text{Var}\left(\frac{1}{4} \sum_{i=1}^4 X_i\right)$$

$$= \frac{1}{4^2} \text{Var}(0.5X_1 + X_2 + 0.5X_3 + 2X_4)$$

$$= \frac{1}{4^2} [\text{Var}(0.5X_1) + \text{Var}(X_2) + \text{Var}(0.5X_3) + \text{Var}(2X_4)]$$

$$= \frac{1}{4^2} [0.25\sigma_x^2 + \sigma_x^2 + 0.25\sigma_x^2 + 4\sigma_x^2]$$

$$= \frac{5.5\sigma_x^2}{16}$$

$$= 0.34\sigma_x^2$$

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ ? Why?

$\bar{X}$ is more efficient estimator of μ_x than $\tilde{X}$ because it has smaller variance.