Integration

Thursday, November 26, 2020 10:38 AM

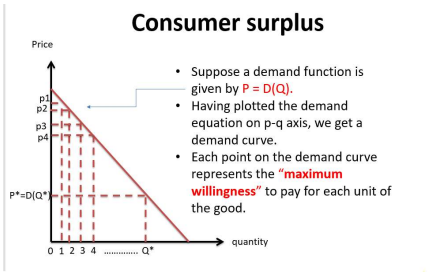
8.3) Applications of integration in economics.

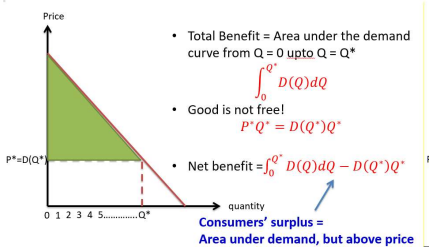
Recoverability: getting back to the level function from the marginal function

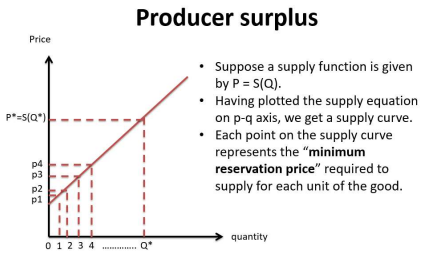
- $MU(x) \rightarrow U(x)$
- $MC(Q) \rightarrow C(Q)$
- Invement as a change in capital stock: $I(t) = \frac{dK(t)}{dt} \rightarrow K(t)$

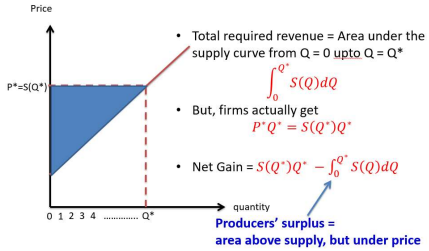
Welfare analysis:

- We normally want to measure the desirability of market outcome or market allocation.
- There are several welfare measurements
- Consumers' and producers' surplus (CS/PS) are the most popularized concept that most economists use.
  - Given information about demand and supply curve, we can measure CS and PS by applying the (definite) integration as the tool for measurement.
- What are they?

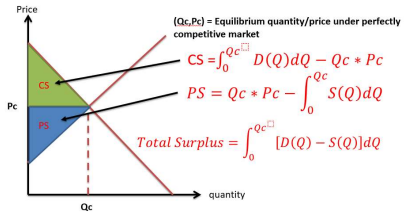








Welfare under perfectly competitive equilibrium

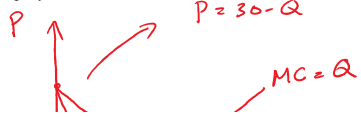


Example: Suppose the market demand equation is $Q = \frac{1}{2}(25 - P)$ and the market supply equation is $Q = \frac{1}{2}(P - 1)$. Evaluate the welfare at the market equilibrium price and quantity.

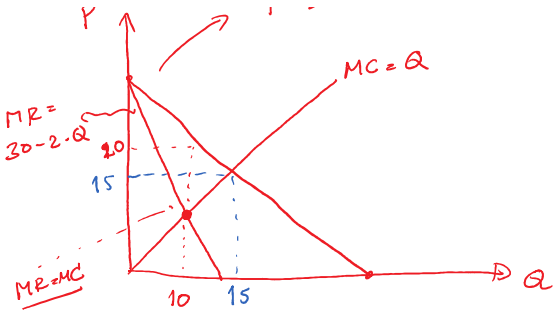
First-best equilibrium:

- Economist typically refers to the "perfectly competitive" equilibrium/allocation as the benchmark for the first-best equilibrium.
  - Proof should be given in Micro311, for the part that discusses about General Equilibrium.
  - Rigorous proof can be found in Debreu (1959) "*Theory of value*".
- In what sense?
  - In the sense that the allocation generates the biggest total surplus of the two (consumers/producers) combined.
- Any market inventions on the perfectly competitive market would do more harm than good.
  - Any interventions, such as taxes/subsidies/regulations, would do harm, rather than doing good, e.g. yielding us lowered welfare
- Any markets with the structure deviating from the assumptions imposed under perfectly competitive market would yield lower welfare.

Example: Suppose that the market demand is given by P = 30 – Q and the cost function is given by $C(Q) = \frac{1}{2}Q^2$. Calculate the Deadweight loss that arises under monopoly market.



⇒ welfare under Monopoly ⇒ Monopoly Eq^m
↳ MR = MC
V.S.
welfare under P.C. ⇒ P.C. Eq^m



V.s.

Welfare under P.C. ⇒ P.C. Eq^{n}

↳ MR = MC

↳ P = MC

$MR = \frac{dTR}{dQ}$

$MC = \frac{dC}{dQ} = Q$

$Q^M \Rightarrow \max \Pi = P \cdot Q - C$

F.O.C. ⇒ $Q^M \Rightarrow MR = MC$

$30 - 2 \cdot Q = Q$

$Q^M \Rightarrow 10$ units

$p^M = 30 - Q^M = 30 - 10 = \$20$

Eq^{n} Perfect ⇒ $P = MC$

$30 - Q = Q \Rightarrow Q^{PC} = 15$ units

$p^{PC} = 30 - Q^{PC}$

$= 30 - 15 = 15\$$

Consume

Monopoly $\int_0^{10} (30-Q)\,dQ - (10)(20)$

$30Q - \frac{Q^2}{2}\Big]_0^{10} \Rightarrow \left(30(10) - \frac{10^2}{2}\right) - \left(30(0) - \frac{0^2}{2}\right)$

$\Rightarrow 300 - 50 \Rightarrow 250$

CS ⇒ 250 − 200 = 50 #

PS ⇒ $(10)(20) - \int_0^{10} (\cdot Q)\,dQ$

$\frac{Q^2}{2}\Big]_0^{10} \Rightarrow \frac{10^2}{2} - \frac{0^2}{2} = 50$

PS = 200 − 50 = 150 #

Welfare

CS + PS

200

Perfect Comp

CS ⇒ $\int_0^{15} (30-Q)\cdot dQ - (15)(15)$

$30Q - \frac{Q^2}{2}\Big]_0^{15} \Rightarrow (30)(15) - \frac{(15)^2}{2} - 0$

$CS = 30(15) - \frac{15^2}{2} - (15)^2$

Calculate number

PS ⇒ $(15)(15) - \int_0^{15} \cdot Q \cdot dQ$

$\frac{Q^2}{2}\Big]_0^{15} \Rightarrow \frac{15^2}{2} - 0$

$PS = (15)^2 - \frac{15^2}{2}$

Welfare = $30(15) - 15^2 - \frac{15^2}{2} + 15^2 - \frac{15^2}{2}$

PW = 25

$\frac{Q^2}{2}\Big]_0^{15} \Rightarrow \frac{15^2}{2} - 0$

Welfare $= 30(15) - \frac{15^2}{2} - 15^2 + 15^2 - \frac{15^2}{2}$

$= 30(15) - (15)^2$

$= 2(15)(15) - (15)^2 \Rightarrow 2 \cdot 15^2 - 15^2$

$\Rightarrow 15^2 = 225$