FN241: Session 4-5
Theory of Risk and Return

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Defining Return and Risk

Return

= Income received on an investment plus any change in market price, usually expressed as a percent of the beginning market price of the investment.

Risk

= The variability of returns from those that are expected.

Key questions ...

What characterizes stock returns?

- Are returns predictable?
- How volatile are stock returns?
- How does volatility change over time?
- What types of stocks have the highest returns?

Are returns predictable?

Capital Asset Pricing Model (CAPM)

- **CAPM** is a model that describes the relationship between **risk and expected (required) return**; in this model, a security's expected (required) return is **the risk-free rate** **plus** **a premium** based on the **systematic risk** of the security.

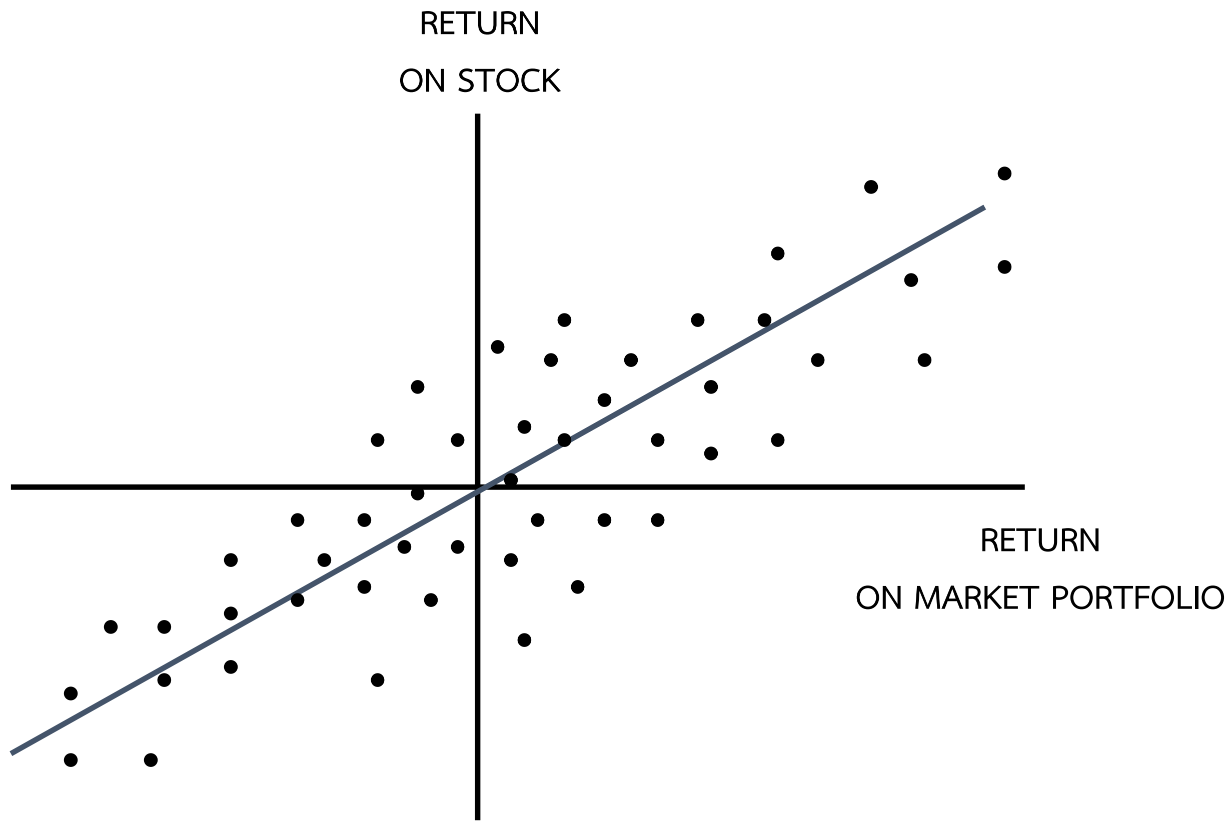
R_j is the required rate of return for stock j,

R_f is the risk-free rate of return,

β_j is the beta of stock j (measures systematic risk of stock j),

R_M is the expected return for the market portfolio.

Characteristic Line

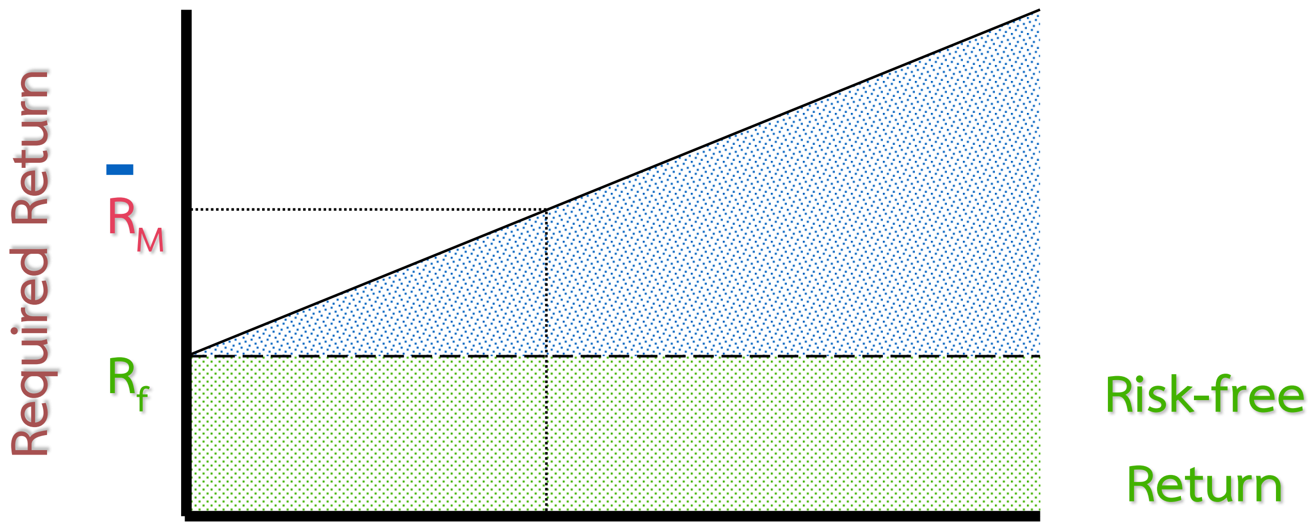


What is Beta?

- An index of systematic risk.
- It measures the sensitivity of a stock's returns to changes in returns on the market portfolio.
- The beta for a portfolio is simply a weighted average of the individual stock betas in the portfolio.

Security Market Line

$$R_j = R_f + \beta_j(R_M - R_f)$$



Security Market Line

- <u>Obtaining Betas</u>
  - Can use <u>**historical data**</u> if past best represents the expectations of the future
  - Can also utilize services like Value Line, Ibbotson Associates, etc.
- <u>Adjusted Beta</u>
  - Betas have a tendency to revert to the mean of 1.0
  - Can utilize combination of recent beta and mean
    - 2.22 (.7) + 1.00 (.3) = 1.554 + 0.300 = <u>1.854 estimate</u>

Is CAPM appropriate in all situations?

Case Study:

How volatile are stock returns?

Definition of Volatility

- Suppose that S_i is the value of a variable on day *i*. The volatility per day is the **standard deviation** of:

or

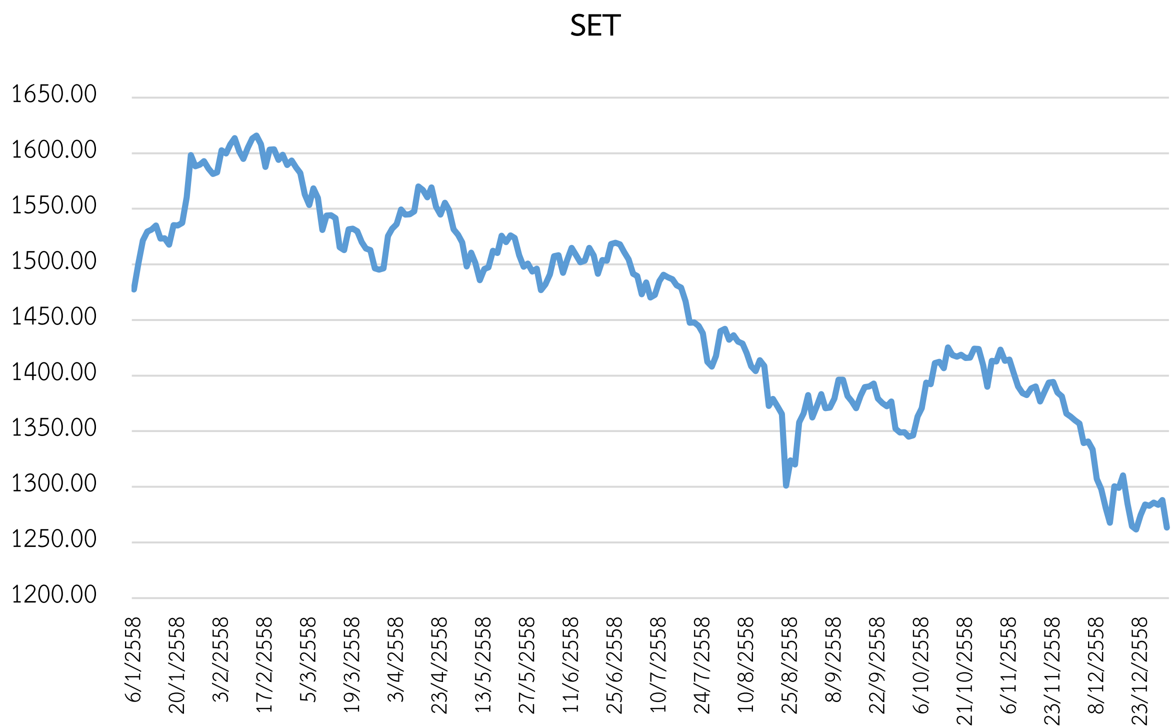
- Normally, **days when markets are closed are ignored** in volatility calculations. Thus, the volatility per year is number of business days times the daily volatility:

or

- **Variance rate** is the square of volatility

Implied Volatilities

- Since volatility cannot be directly observed, we can therefore imply volatilities from **market prices** and vice versa



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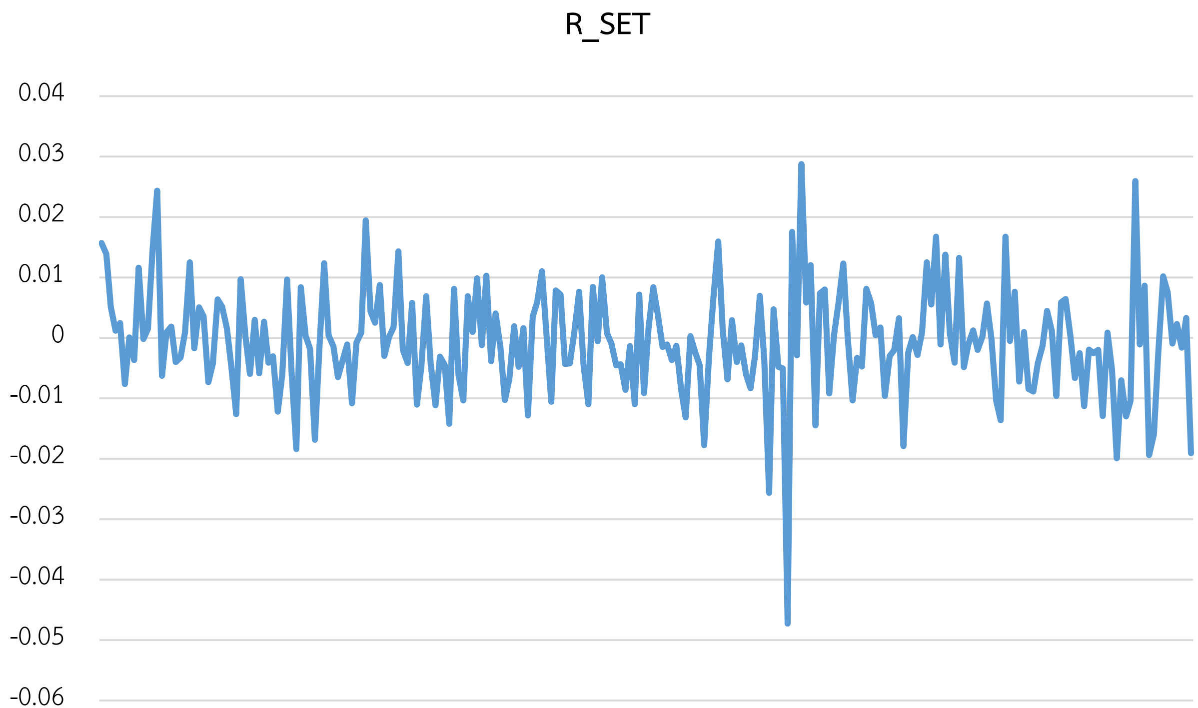
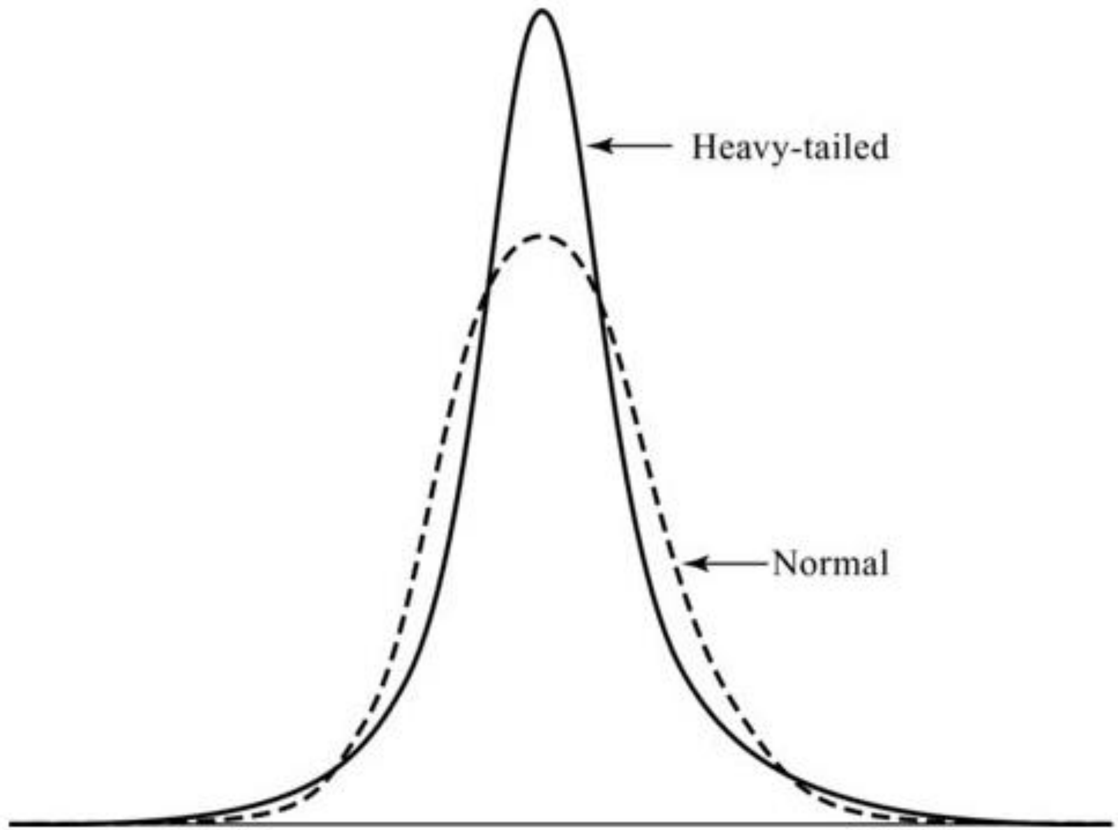


Heavy Tails

- Daily returns or changes are **NOT normally distributed**
  - The distribution has **heavier tails** than the normal distribution
  - It is **more peaked** than the normal distribution
- This means that **small changes and large changes are more likely** than the normal distribution would suggest
- Many market variables have this property, known as **EXCESS KURTOSIS.**

Heavy-Tailed vs. Normal Distribution



Alternatives to Normal Distributions: The Power Law

- An alternative to assuming normal distributions.
- It is approximately true that the value of the variable, *v*, has the property that when *x* is large:

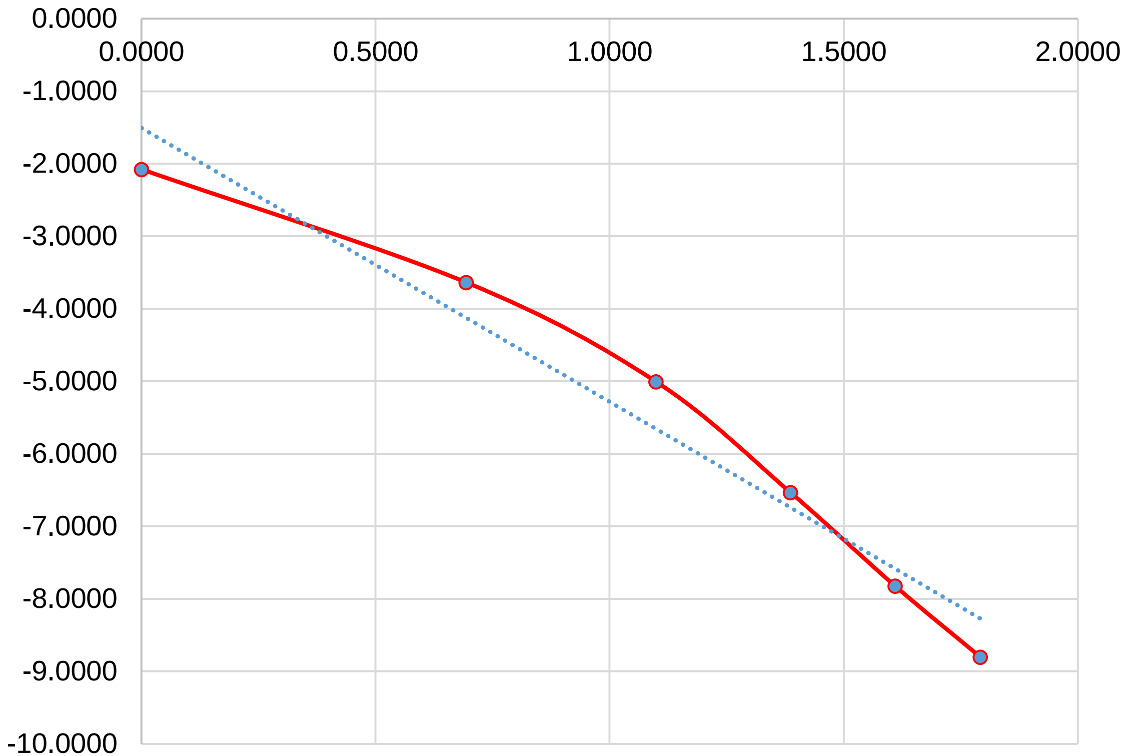
$$Prob(v > x) = Kx^{-\alpha}$$

- This seems to fit the behavior of the returns on many market variables better than the normal distribution

Alternatives to Normal Distributions: The Power Law

	State	Real World (%)	Normal Model (%)	ln(x)	Prob(v>x)	ln[Prob(v>x)]
>1 SD	1	25.04	31.73	0.0000	**0.1252**	**-2.0778**
>2SD	2	5.27	4.55	0.6931	**0.02635**	**-3.6363**
>3SD	3	1.34	0.27	1.0986	**0.0067**	**-5.0056**
>4SD	4	0.29	0.01	1.3863	**0.00145**	**-6.5362**
>5SD	5	0.08	0	1.6094	**0.0004**	**-7.8240**
>6SD	6	0.03	0	1.7918	**0.00015**	**-8.8049**

Alternatives to Normal Distributions: The Power Law



What are K and alpha?

How does volatility change over time?

Standard Approach to Estimating Volatility

- Define σ_n as the volatility per day (on day n) between day n-1 and day n, as estimated at end of day n-1
- Define S_i as the value of market variable at end of day i
- Define

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right)$$

then, we have

$$\sigma_n^2 = \frac{1}{m-1}\sum_{i=1}^{m}(u_{n-i} - \bar{u})^2$$

$$\bar{u} = \frac{1}{m}\sum_{i=1}^{m}u_{n-i}$$

Standard Approach to Estimating Volatility

- Simplifications Usually Made in Risk Management
  - Define
  - Assume that the mean value of u_i is zero
  - Replace m-1 by m
- This gives:

Weighting Scheme

Instead of assigning equal weights to the observations we can set

$$\sigma_n^2 = \sum_{i=1}^{m} \alpha_i u_{n-i}^2 \quad \text{where} \quad \sum_{i=1}^{m} \alpha_i = 1$$

In an ARCH(m) model we also **assign some weight to the long-run variance rate**, V_L:

where

Generalized Autoregressive Conditional Heteroskedasticity: GARCH (p,q)

Note for general form of the Generalized Autoregressive Conditional Heteroskedasticity:

$$\boxed{\sigma_n^2 = \omega + \sum_{i=1}^{p} \alpha_i u_{n-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{n-j}^2}$$

GARCH (1,1)

- In GARCH (1,1) we assign some **weight to the long-run average variance rate**

$$\boxed{\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2}$$

Since weights must sum to 1:

Setting $\omega = \gamma V_L$ the GARCH (1,1) model is

and

GARCH (1,1)

Example: Suppose

$$\sigma_n^2 = 0.000002 + 0.13u_{n-1}^2 + 0.86\sigma_{n-1}^2$$

What are the long-run variance rate and the long-run volatility per day?

Suppose that the estimate of the volatility on day is 1.6% per day and that on day n-1 the market variable decreases by 1%, what is the new estimated volatility?

Forecasting Future Volatility

A few lines of algebra shows that:

$$
\begin{aligned}
\sigma_n^2 &= (1-\alpha-\beta)V_L + \alpha u_{n-1}^2 + \beta\sigma_{n-1}^2 \\
\sigma_n^2 - V_L &= \alpha(u_{n-1}^2 - V_L) + \beta(\sigma_{n-1}^2 - V_L) \\
\sigma_{n+t}^2 - V_L &= \alpha(u_{n+t-1}^2 - V_L) + \beta(\sigma_{n+t-1}^2 - V_L) \\
E\left[\sigma_{n+t}^2 - V_L\right] &= (\alpha+\beta)E\left[\sigma_{n+t-1}^2 - V_L\right] \\
E\left[\sigma_{n+t}^2 - V_L\right] &= (\alpha+\beta)E\left[\sigma_{n+t-1}^2 - V_L\right]
\end{aligned}
$$

To estimate the volatility for an option lasting T days we must integrate this from 0 to T

Volatility Term Structures

Assume that

$$V(t) = E[\sigma_{n+t}^2] \qquad \text{and} \qquad a = \ln \frac{1}{\alpha + \beta}$$

Thus $E[\sigma_{n+t}^2] = V_L + (\alpha + \beta)^t (\sigma_n^2 - V_L)$ becomes:

$$V(t) = V_L + e^{-at}\left[V(0) - V_L\right]$$

The average variance rate per day between today and time *T* is:

$$\frac{1}{T}\int_0^T V(t)dt = V_L + \frac{1-e^{-aT}}{aT}\left[V(0) - V_L\right]$$

Assuming 252 business days per year, the variance rate per day is?

Forecasting Future Volatility

The volatility per year lasting T days is

$$\boxed{\sigma(T) = \sqrt{252\left\{V_L + \frac{1-e^{-aT}}{aT}\left[V(0) - V_L\right]\right\}}}$$

where $$a = \ln \frac{1}{\alpha + \beta}$$

Forecasting Future Volatility

- Suppose that GARCH(1,1) parameters have been estimated as w= 0.000003, alpha=0.04, and beta=0.94. The current daily volatility is estimated to be 1%. Estimate the daily volatility in 30 days.
- Suppose that GARCH(1,1) parameters have been estimated as w= 0.000002, alpha=0.04, and beta=0.94. The current daily volatility is estimated to be 1.3%. Estimate the volatility per annum that should be used to price a 20-day option.

What types of stocks have the highest returns?

Traditional Performance Measures

- Sharpe Measure
- Treynor Measures
- Jensen Measure
- Performance Measurement in Practice

Sharpe and Treynor Measures

Sharpe measure =

Treynor measure =

where $\overline{R} =$ average return

$R_f =$ risk-free rate

$\sigma =$ standard deviation of returns

$\beta =$ beta

- The **Sharpe measure** evaluates return relative to total risk
  - Appropriate for a well-diversified portfolio, but not for individual securities
- The **Treynor measure** evaluates the return relative to beta, a measure of systematic risk
  - It ignores any unsystematic risk

Example: Sharpe and Treynor Measures

- Over the last four months, XYZ Stock had excess returns of 1.86 percent, –5.09 percent, –1.99 percent, and 1.72 percent. The standard deviation of XYZ stock returns is 3.07 percent. XYZ Stock has a beta of 1.20.
  - What are the Sharpe and Treynor measures for XYZ Stock?

Jensen Measure

- The Jensen measure stems directly from the CAPM:

- The constant term should be zero
  - Securities with a beta of zero should have an excess return of zero according to finance theory
- According to the Jensen measure, if a portfolio manager is better-than-average, the **alpha** of the portfolio will be positive

Academic Issues Regarding Performance Measures

- The use of Treynor and Jensen performance measures relies on measuring the market return and CAPM
  - Difficult to identify and measure the return of the market portfolio
- Evidence continues to accumulate that may ultimately displace the CAPM
  - Arbitrage pricing model, multi-factor CAPMs, inflation-adjusted CAPM

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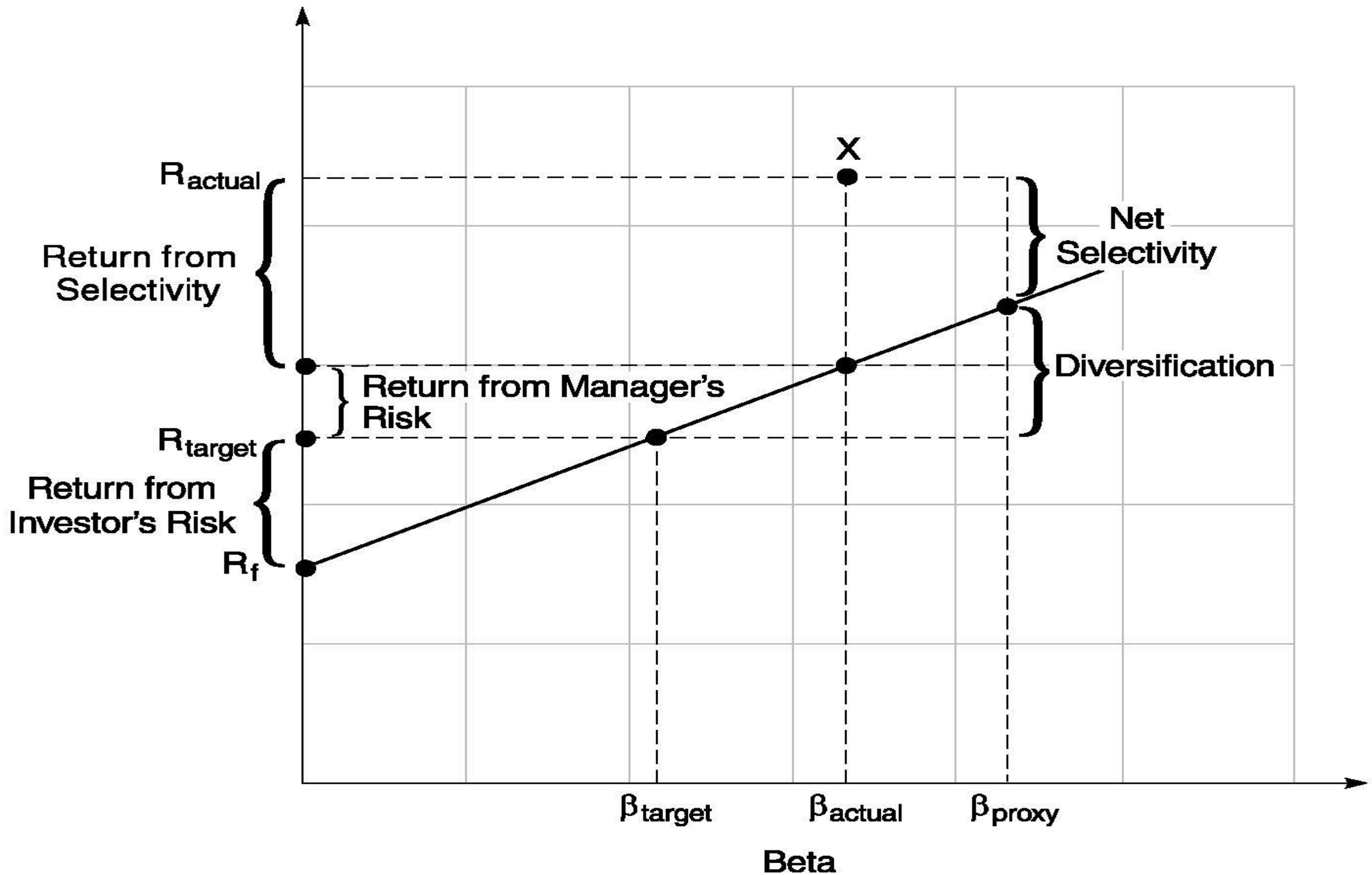
Industry Issues

- "Portfolio managers are hired and fired largely on the basis of realized investment returns with little regard to risk taken in achieving the returns"
- Practical performance measures typically involve a comparison of the fund's performance with that of a benchmark
- "Fama's return decomposition" can be used to assess why an investment performed better or worse than expected:
    - The return the investor chose to take
    - The added return the manager chose to seek
    - The return from the manager's good selection of securities



Source: Adapted from Eugene Fama, "Components of Investment Performance," *Journal of Finance*, June 1972, 551–567.

Industry Issues

- Diversification is the difference between the return corresponding to the beta implied by the total risk of the portfolio and the return corresponding to its actual beta
  - Diversifiable risk decreases as portfolio size increases, so if the portfolio is well diversified the “diversification return” should be near zero
- Net selectivity measures the portion of the return from selectivity in excess of that provided by the “diversification” component

Dollar-Weighted and Time-Weighted Rates of Return

- The **dollar-weighted rate of return** is analogous to the internal rate of return in corporate finance
  - It is the rate of return that makes the present value of a series of cash flows equal to the cost of the investment:

Dollar-Weighted and Time-Weighted Rates of Return

- The **time-weighted rate of return** measures the compound growth rate of an investment
  - It eliminates the effect of cash inflows and outflows by computing a return for each period and linking them (like the geometric mean return):

Note: The time-weighted rate of return and the dollar-weighted rate of return will be equal if there are no inflows or outflows from the portfolio

Question?