Answer

1. Given : Demand $Q = 10000 + 400(T-30) - 10P$

Supply $Q = 2000 + 20P$

If $T = 40 \Rightarrow Q^d = 10000 + 400(40-30) - 10P$

$Q^d = 14000 - 10P \Rightarrow Q + 10P = 14000$

$Q^s = 2000 + 20P \Rightarrow Q - 20P = 2000$

Matrix form

$$x = \begin{bmatrix} Q \\ P \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 10 \\ 1 & -20 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} Q \\ P \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 14000 \\ 2000 \end{bmatrix}}_{d}$$

$$x = A^{-1} d$$

$$A^{-1} = \frac{adj(A)}{det(A)}$$

$$adj(A) = [Cof(A)]^T$$

$$Cof(A) = \begin{bmatrix} -20 & -1 \\ -10 & 1 \end{bmatrix} \Rightarrow [Cof(A)]^T = \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix}$$

$$det(A) = 1(-20) - 10 = -30$$

$$A^{-1} = \frac{1}{-30} \begin{bmatrix} -20 & -10 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 \\ 1/30 & -1/30 \end{bmatrix}$$

Solve for $x^* = \begin{bmatrix} Q^* \\ P^* \end{bmatrix}$

$$x^* = A^{-1} d$$

$$\begin{bmatrix} Q^* \\ P^* \end{bmatrix} = \begin{bmatrix} 2/3 & 1/3 \\ 1/30 & -1/30 \end{bmatrix} \begin{bmatrix} 14000 \\ 2000 \end{bmatrix} = \begin{bmatrix} 9{,}333.33 + 666.67 \\ 466.67 - 66.67 \end{bmatrix} = \begin{bmatrix} 10000 \\ 400 \end{bmatrix}$$

There fore $Q^* = 10{,}000$ unit , $P^* = 400$ unit

Answer

2\. a. The assumption over the behavior of government does not make any sense in practice because in real life GDP consists of all type of expenditures including individual consumption , public spending , and private spending ; therefore, a country 's GDP should not be tied with government spending.

b C_2 represents the marginal propensity to consume depending on nominal interest rate. It implies that if the interest rate increases, the consumption of individuals fall which may be due to individual cut consumption to saving.

c. Derive IS equation :

$$Y = C + I + G$$

$$= C_0 + C_1 Y_d - C_2 r + G_0 - G_1 Y + I_0 + I_1 Y - I_2 r$$

$$= C_0 + C_1 (Y - T_0) - C_2 r + G_0 - G_1 Y + I_0 + I_1 Y - I_2 r$$

$$= C_0 + C_1 Y - C_1 T_0 - C_2 r + G_0 - G_1 Y + I_0 + I_1 Y - I_2 r$$

$$Y = C_0 - C_1 T_0 + G_0 + (C_1 - G_1 + I_1) Y - (C_2 + I_2) r$$

$$Y - (C_1 - G_1 + I_1) Y = C_0 + G_0 - C_1 T_0 - (C_2 + I_2) r$$

$$Y [1 - (C_1 - G_1 + I_1)] = C_0 + G_0 - C_1 T_0 - (C_2 + I_2) r \Rightarrow Y = \frac{C_0 + G_0 - C_1 T_0 - (C_2 + I_2) r}{[1 - (C_1 - G_1 + I_1)]} \ \#$$

Interpret the derived IS equation

From IS equation that had been derived above, we can see the negative relationship between income or GDP (Y) and interest rate (r). If interest rate is high, investment and consumption will decrease . ($-(C_2 + I_2)$ is negative slope of r).

d. Calculate the slope of IS curve.

Since IS equation : $Y [1 - (C_1 - G_1 + I_1)] = C_0 + G_0 - C_1 T_0 - (C_2 + I_2) r$

$$\Rightarrow \quad r = \frac{C_0 + G_0 - C_1T_0 - Y[1-(C_1-G_1+I_1)]}{(C_2+I_2)}$$

$\Rightarrow$ The slope of IS curve is $-[1-(C_1-G_1+I_1)]$. #

IS curve flat when investment and consumption are sensitive to change in interest rate.

The flat IS curve implies that small changes in interest rate (R) will make big changes in GDP or income (Y).

e. Find tax multiplier.

IS equation : $Y[1-(C_1-G_1+I_1)] = C_0 + G_0 - C_1T_0 - (C_2+I_2)r$

$$Y = \frac{1}{1-(C_1-G_1+I_1)} C_0 + G_0 - C_1T_0 - (C_2+I_2)r$$

$$\frac{\partial Y}{\partial T_0} = \frac{-C_1(1-(C_1-G_1+I_1))}{[1-(C_1-G_1+I_1)]^2} = \frac{-C_1}{1-C_1+G_1-I_1} \quad \#$$

The tax multiplier depends on slope of IS curve because if the slope IS curve is large, the tax multiplier will be small, which it further makes the output change a littler due to small tax multiplier.

f. Derive LM equation.

LM equation : $M_s = L^d$

$$\Rightarrow \quad M_0 = L_0 + L_1Y - L_2r$$

$$L_1Y = M_0 - L_0 + L_2r$$

$$Y = \frac{M_0 - L_0}{L_1} + \frac{L_2}{L_1}r \quad \#$$

From the LM equation that has been derived above, we can see that interest rate and GDP has positive relationship. For intance, if interest rate goes up, output or GDP will increase and vice versa.

g. Calculate the slope of LM curve

LM equation : $Y = \frac{M_0 - L_0}{L_1} + \frac{L_2}{L_1} r$

$$\Rightarrow r = \left(Y - \frac{M_0 - L_0}{L_1}\right) \times \frac{L_1}{L_2}$$

$$r = \frac{L_1}{L_2} Y - \frac{M_0 - L_0}{L_2}$$

$\Rightarrow$ slope of LM curve is $\frac{L_1}{L_2}$. #

Flat LM occurs when L_2 is large which means money demand (M_d) is sensitive to change in interest rate. So, if interest rate changes, M_d will differ a lot which makes output (Y) adjust a lot too to hold the equilibrium in money market ($M_d = M_s$).

h. Write IS and LM curve in matrix form.

We have
$$\begin{cases} Y = \frac{M_0 - L_0}{L_1} + \frac{L_2}{L_1} r & \text{(IS equation)} \\ Y = \frac{C_0 + G_0 - C_1 T_0 - (C_2 + I_2) r}{[1 - (C_1 - G_1 + I_1)]} & \text{(LM equation)} \end{cases}$$

$$\Rightarrow \begin{cases} L_1 Y - L_2 r = M_0 - L_0 \\ Y[1 - (C_1 - G_1 + I_1)] + (C_2 + I_2) r = C_0 + G_0 - C_1 T_0 \end{cases}$$

Matrix form :

$$\underset{(A)}{\begin{bmatrix} L_1 & L_2 \\ [1 - (C_1 - G_1 + I_1)] & C_2 + I_2 \end{bmatrix}} \underset{(x)}{\begin{bmatrix} Y \\ r \end{bmatrix}} = \underset{(d)}{\begin{bmatrix} M_0 - L_0 \\ C_0 + G_0 - C_1 T_0 \end{bmatrix}}$$

i : Solve for equilibrium Y^*, r^* using Cramer's rule

$$Y^* = \frac{\det(A_y)}{\det(A)} \quad \text{and} \quad r^* = \frac{\det(A_r)}{\det(A)}$$

$$\det(A) = L_1 (C_2 + I_2) - [1 - (C_1 - G_1 + I_1)] L_2$$

$$A_y = \begin{bmatrix} M_0 - L_0 & L_2 \\ C_0 + G_0 - C_1 T_0 & C_2 + I_2 \end{bmatrix} \Rightarrow \det(A_y) = (M_0 - L_0)(C_2 + I_2) - L_2 (C_0 + G_0 - C_1 T_0)$$

$$\Rightarrow \quad Y^* = \frac{(M_0-L_0)(C_2+I_2) - L_2(C_0+G_0-C_1T_0)}{L_1(C_2+I_2) - [1-(C_1-G_1+I_1)]L_2}$$

$$A_r = \begin{bmatrix} L_1 & M_0 - L_0 \\ [1-(C_1-G_1+I_1)] & C_0+G_0-C_1T_0 \end{bmatrix} \Rightarrow \det(A_r) = L_1(C_0+G_0-C_1T_0) - (M_0-L_0)[1-(C_1-G_1+I_1)]$$

$$\Rightarrow \quad r^* = \frac{L_1(C_0+G_0-C_1T_0) - (M_0-L_0)[1-(C_1-G_1+I_1)]}{L_1(C_2+I_2) - [1-(C_1-G_1+I_1)]L_2} \quad \#$$

j. Calculate multiplier of G_0 and the multiplier of M_0 on both Y^* and r^*, respectively.

find multiplier G_0 of Y^*

$$\frac{\partial Y^*}{\partial G_0} = \frac{-L_2C_0 - L_2 + C_1T_0L_2}{L_1(C_2+I_2) - [1-(C_1-G_1+I_1)]L_2} = \frac{-L_2(C_0+1-C_1T_0)}{L_1(C_2+I_2) - [1-(C_1-G_1+I_1)]L_2}$$

$$\frac{\partial Y^*}{\partial M_0} = \frac{C_2+I_2}{L_1(C_2+I_2) - [1-(C_1-G_1+I_1)]L_2}$$

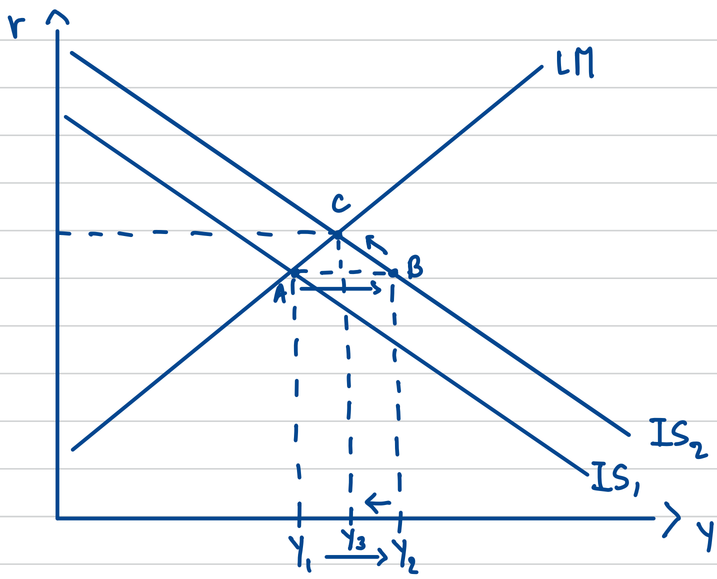
find multiplier M_0 of r^*

$$\frac{\partial r^*}{\partial M_0} = \frac{-(M_0+C_1M_0+G_1M_0-I_1M_0)}{L_1(C_2+I_2) - [1-(C_1-G_1+I_1)]L_2} = \frac{-(1+C_1+G_1-I_1)}{L_1(C_2+I_2) - [1-(C_1-G_1+I_1)]L_2}$$

$$\frac{\partial r^*}{\partial G_0} = \frac{L_1}{L_1(C_2+I_2) - [1-(C_1-G_1+I_1)]L_2}$$

If the government spending is purely exogenous, i.e $G_1=0$, the multipler will be bigger than the government spending that is endogenous.

K. The government cannot successfully achieve both goals by simply relying on a single type of policy implemented. This is because these two goals are trade-off to each other; that is, to increase GDP by increasing government spending, there will be higher interest rate.



c. To achieve both goal simultaneously, government can use expansionary fiscal policy and expansionary monetary policy. By increasing government expenditure, GDP will be increased, and by increasing money supply, interest rate will be lowered.