EE320
semester 2/2015

Lecture 4 Introduction to Mathematical Economic Applications
Read chapter 2

Economic Models: 4 Applications

- **Profit maximization of a competitive firm**: topic 2.3.
- **Profit maximization of a monopoly and taxation**: topic 2.4.
- **Profit maximization of a duopoly:** topic 2.5
- **A simple macroeconomic model**: topic 2.7

1. Profit max for a competitive firm

- **Environment**: Each firm is a price taker in both output and input markets: no control over the price at which it can sell its product, and no control over the prices that must be paid for inputs. Thus, all market prices are …?….genous variables when making decisions.
- **Choices** to be made are: inputs and output
- **Goal**: to maximize its profits.
- **Assumptions**:
  – using only 2 inputs (K capital and L labor)
  – Technology is given by $Q = 2\sqrt{KL} = 2K^{0.5}L^{0.5}$
  – Short-run: capital is fixed

- To remind yourself that K is fixed, we use subscript 0 for K: K_0
- We can write down profits as:

$$\pi = PQ - wL - rK_0 \quad \text{..................(1)}$$

- Where P is output price, w wage rate, r price of capital.
- What are endogenous variables? ….., …..
- From our production function, if we choose L and also determine Q. We can rewrite (1) by substitute Q in (1) as

$$\pi = P2\sqrt{K_0L} - wL - rK_0 \quad \text{.........(2)}$$

- FONC wrt L : $$\boxed{\frac{d\pi}{dL} = PK_0^{0.5}L^{-0.5} - w = 0.}$$

- Meaning: marginal benefit = marginal cost from an increase in one unit of labor.
- RHS called "value of marginal product of labor".

- SOC:

$$\frac{d^2\pi}{dL^2} = -0.5PK_0^{0.5}L^{-1.5} \quad < \quad 0$$

- Since SOC holds, we can solve for L from FOC.

$$L^* = \frac{P^2}{w^2}K_0$$

- We get the solution L as a function of exogenous variables.
- How do we call this function?. ……………………..

- Next, you can do some comparative static analysis of the equilibrium level of L.
- Try taking the derivatives wrt P : sign ……..
- Taking the derivative wrt K_0 : sign …………….
- Taking the derivative wrt w : sign ……………
- Explain : What happen to marginal product of labor (MPL) when firms get more K or sell at higher price? how this change value of MPL? ……………..
- So, firms should hire ……… of labor.
- Explain: when wage increases, …………………

- We can find the optimal value of output since we know L*.
- Putting L* into the production function, you get

$$Q^* = 2K_0^{0.5}(L^*)^{0.5} = 2K_0^{0.5}\left(\frac{P^2K_0}{w^2}\right) = \frac{2PK_0}{w}.$$

- We call this "………………………………………………".
- You can check if this …………. function behaves as you know.
- In sum, profit-max problem of a competitive firm give us (1) …Demand for ………………… and
  (2) ………………………………………………………

2. Profit maximization of a monopoly

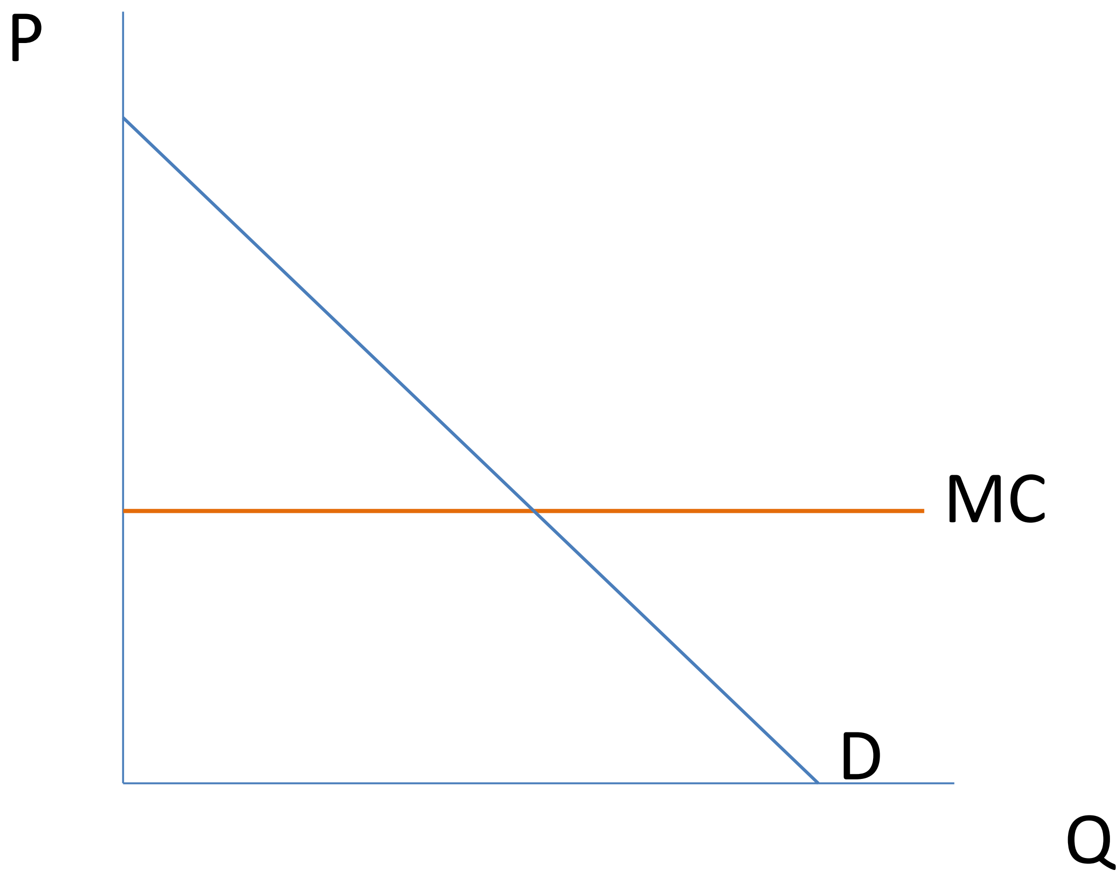
- Key: the demand faced by a firm is identical to the market demand curve.
- Assumptions:
  - linear demand curve: $P = a - b\,Q$
  - constant marginal production cost, c. The total cost function is $f + cQ$, where f is fixed cost.
- Goal: maximize profits

$$\pi = TR - TC = PQ - f - cQ$$

- Choices: output level, Q or price level P.
- But both variables (output and price) are dependent, so we can choose just one.
- Suppose we select Q and write profits as a function of Q:

$$\pi = (a - bQ)Q - f - cQ = (aQ - bQ^2) - f - cQ$$

- FOC: $\dfrac{d\pi}{dQ} = \dfrac{dTR}{dQ} - \dfrac{dTC}{dQ} = MR - MC = (a - 2bQ) - c \;\; = \;\; 0.$



Draw MR, check slope of MR =.................. and vertical intercept is ....

- SOC: $\dfrac{d^2\pi}{dQ^2} = \qquad < \ 0.$
- We can find equilibrium from FOC:
- P* =
- Q* =
- What do we require for *a* and *c* ?
- Remember pricing rule of monopolist: mark-up over its marginal cost
- Noting two issues: fixed costs do not affect the decision at the margin. But fixed costs can affect profits if f is large so that maximized profit can be negative.

- Suppose the government imposes a per-unit tax on the monopolist's output.
- Total taxes : $T = tQ$
- Profit after tax,
- $\pi = TR \ - TC = PQ - tQ - cQ - f$
- $\pi = (a - bQ)Q - tQ - cQ - f$
- $\quad = (aQ - bQ^2) - (c + t)Q - f$
- FOC:
- SOC:
- Meaning: new MC = MC + tax

$$Q^* = \frac{(a - c - t)}{2b}, \quad P^* = \frac{(a + c + t)}{2}.$$

- Old trick also works here.
- We can find the optimal profits by putting Q* and P* into the profit function.

$$\pi^* = \frac{(a - c - t)^2}{4b} - f.$$

- We can do comparative static analyses by taking derivatives:

$$\frac{\partial Q^*}{\partial t} = \qquad\qquad < 0, \quad \frac{\partial P^*}{\partial t} = \qquad\qquad > 0, \quad \frac{\partial \pi^*}{\partial t} = \qquad\qquad < 0.$$

- How much consumers pay more due to taxation on a monopolist.
- Does tax always reduce the monopolist's profits?
- If we ask about the government's decision on the level of the tax rate. Assuming trying to maximize total tax revenue.
- What is the optimal tax rate for the government?
- We need to make t as endogenous variable
- Tax revenue,

$$T = tQ^* = t\left(\frac{a-c-t}{2b}\right)$$

$$FOC: \quad \frac{dT}{dt} = \frac{a-c-2t}{2b} = 0,$$

$$SOC: \quad \frac{d^2T}{dt^2} = \frac{-1}{b} < 0.$$

$$thus,\ t^* = \frac{a-c}{2}.$$

3. Profit maximization of a duopoly

- Two firms competing: firm 1 and firm 2
- Cournot fashion (output competition): decide q_1 and q_2 at the same time.
- Assumptions: 1. Linear market demand; 2. producing identical products; 3. linear total cost and constant marginal cost; 4. no fixed cost; 5. both use the same technology and same marginal cost.
- Profit function for each firm is identical, just a different subscript $i = 1, 2$

$$\pi_1 = TR_1 - TC_1 = Pq_1 - cq_1 = (a - bQ)q_1 - cq_1$$

$$= (a - b(q_1 + q_2))q_1 - cq_1$$

$$=$$

$$=$$

$$\pi_2 \qquad =$$

- Endogenous variables are ……………………………
- Exogenous or parameters are ………………………
- Assume: each firm treats the other firm's output as an exogenous variable or given.
- Each firm tries to find his best response (selecting output level) to opponent's output.
- Equilibrium in this model is called Nash Equilibrium
- NE: each player chooses the best action he can, given other player's action.

- For firm 1: FOC

$$\frac{d\pi_1}{dq_1} = (a - bq_2 - 2bq_1) - c = 0, \; and$$

$$\frac{d^2\pi}{dq_1} = -2b < 0.$$

$$\text{Rewrite } FOCs\, as \quad q_1 = \frac{(a-c)}{2b} - \frac{1}{2} q_2 , \; and$$

$$q_2 = \frac{(a-c)}{2b} - \frac{1}{2} q_1 \; .$$

$$this\, equation\, is\, called\, "Best\; reaction\;\; function".$$

$$Solve\, both\, FOCs\;\; for\, q_1^* \;\; and \;\;\; q_2^*$$

HW2: question 1

- 1. solve for solutions in the Cournot duopoly (previous example)
- 2. draw both reaction functions: Let q1 be an horizontal axis, q2 be a vertical axis.
- 3. find the equilibrium outputs and market price
- 4. find both firm's profits
- 5. thinking about some comparative static analyses.

Question 2: read 4. A simple macroeconomic model, topic 2.7

- Accounting identity: GDP = total spending

$$Y = C + I + G$$

- CONSUMPTION: $C = C_0 + bY^d$

where C_0 and b are parameters, and Y^d is after-tax income

- Assume: b is between 0 and 1, as MPC
- Assume: taxes are lump sum as T,
- Thus, $Y^d = Y – T$ and $C = C_0 + b(Y – T)$

- Investment: $I = I_0 - er$

where I_0 and e are positive parameters and r is exogenous variable (in this model, later we will study IS-LM model where r is an endogenous variable)

- Government: G is an exogenous variable
- So, $Y = C_0 + b(Y – T) + I_0 - er + G$
- Only the level of GDP is endogenous
- Find the equilibrium level of GDP?
- Find T and G multipliers? Explain.