

EE431/438 Economics of Financial Markets and
Institutions

Supplementary Solution Exercise 1

Question 3. The lagrangian method.

$$\underset{c_1, c_2}{Max} U(C_1, C_2) = U(C_1) + \beta U(C_2),$$

$$\text{s.t. } C_2 = (1+r)(Y_1 - C_1) + Y_2, \text{ or } -(1+r)(Y_1 - C_1) - Y_2 + C_2 = 0$$

$$\underset{c_1, c_2}{Max} L = U(C_1) + \beta U(C_2) - \lambda(-(1+r)(Y_1 - C_1) - Y_2 + C_2)$$

F.O.C.

$$(1) \frac{\partial L}{\partial C_1} = 0; U'(C_1) = \lambda(1+r)$$

$$(2) \frac{\partial L}{\partial C_2} = 0; \beta U'(C_2) = \lambda$$

$$(3) \frac{\partial L}{\partial \lambda} = -(1+r)(Y_1 - C_1) - Y_2 + C_2 = 0$$

$$(4) \frac{(2)}{(1)} = \frac{U'(C_1)}{\beta U'(C_2)} = (1+r)$$

If we have a specific form of the utility function, (4) shows the relationship between C_1 and C_2 . Then, there are 2 equations (equation (3) and equation (4)) and the two unknowns (C_1, C_2). Thus, we can solve the system of equations to get (C_1^*, C_2^*) .